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Dynamic Equity Slope*

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Abstract

This paper empirically documents that expected growth volatility is a key driver of the equity term structure dynamics. A general equilibrium model jointly explains four important patterns: (i) a potentially negative unconditional equity term premium, (ii) countercyclical equity term premia, (iii) procyclical equity yields, and (iv) premia to value and growth claims respectively increasing and flat with the horizon. The economic mechanism hinges on the interaction between heteroscedastic long-run growth—which leads to countercyclical risk premia—and homoscedastic short-term shocks under limited market participation—which produce sizable risk premia to short-term cash flows. The equity slope dynamics hold irrespective of the sign of its unconditional average.

JEL Classification: D51, D53, E30, G10, G12

Keywords: Term Structure of Equity, Price Dynamics, General Equilibrium, Expected Growth Volatility

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Disclosure Statement

The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

I Introduction

Over the last decade, there has been a significant interest in the term structure of equity, which is a powerful tool to understand equity markets and their connection with economic fundamentals. Pioneering this literature, van Binsbergen, Brandt, and Koijen (2012, henceforth BBK12) show that short-term claims to equity payouts have a higher risk premium than long-term claims, which is at odds with leading asset pricing models. Whereas recent contributions provide a number of potential explanations for this pattern, van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and Gormsen (2021) document rich *conditional dynamics* of the equity term structure, which are still unexplained. These dynamics and their link with economic fundamentals are important because they help understand which risks drive asset price fluctuations. Moreover, the conditional equity term structure is informative about the economic outlook and discount rates and, thus, has implications for real decisions.[1]

This paper documents that expected growth volatility is a key driver of the equity term structure dynamics and proposes a general equilibrium model that explains the most important properties of the equity term structure, including (i) a potentially negative unconditional equity term premium, (ii) procyclical equity yields, (iii) countercyclical equity term premium, (iv) premia to claims of value (respectively, growth) payouts that are increasing (flat) with the horizon, and (v) a countercyclical value premium. Our model links the dynamics of the equity term structure to the timing of risk of economic fundamentals and, specifically, sheds light on the pivotal effect played by expected growth volatility. We provide supportive empirical evidence of the model assumptions and predictions.

The model mechanism hinges on the interaction of two risk factors steering economic fundamentals. The first one is permanent and is driven by time-varying expected growth. It gives rise to a stochastic trend, and induces upward-sloping risk with the horizon. The second

[1]For instance, Gormsen and Koijen (2020) investigate the impact of Covid-19 pandemic on economic growth expectations. Breugem, Marfè, and Zucchi (2021) study the effect of heterogeneity in the pricing and firm's exposure to risks of various persistence on corporate policies and their horizon. Callen and Lyle (2020) estimate the term structure of implied costs of equity capital at the firm level and link it to firm characteristics and performance.

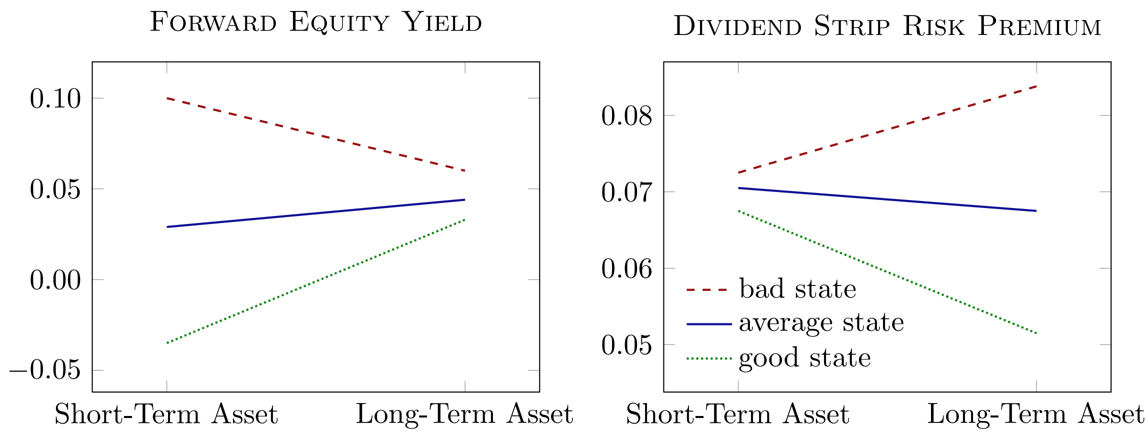

Figure 1: Equity Slope Dynamics. This figure displays the model-implied dynamics of the forward equity yields and the dividend strip risk premium. The short-term and the long-term assets represent the average value of the above quantities respectively over the first five years and the residual infinite-horizon. The state refers to the level of expected growth volatility and, endogenously, to the price level.

one is transitory and produces stationary (short-term) fluctuations, then inducing downward-sloping risk with the horizon. The upward-sloping risk component is heteroscedastic, whereas the downward-sloping one is homoscedastic, as suggested by the data. In equilibrium, the relative weight of these two risks determines the slope of equity compensation across the horizon. To generate a counter-cyclical equity term premium, the weight of the two risk factors needs to be time-varying, with most of the variation driven by the upward-sloping risk factor. Thus, when expected growth volatility (henceforth, EGV) rises, prices decline and become more volatile, short-term equity yields rise relative to long-term ones, and long-term equity risk premia rise relative to short-term ones. Figure 1 summarizes the model predictions about these dynamics. Thus, the model jointly explains the cyclical patterns of the equity term structure in light of macroeconomic risk—endogenizing the findings of van Binsbergen et al. (2013) and Gormsen (2021)—and finds support in our empirical analysis.

We also study the cross-sectional predictions of the model. Heterogeneous loadings on EGV lead to a cross section of equities, whose valuation ratios and risk premia can either decrease or increase with EGV. The model generates a positive and countercyclical value premium (Guo, Savickas, Wang, and Yang, 2009). This result arises jointly with the higher payout cyclicality of value firms relative to growth firms (Koijen, Lustig, and Van Nieuwerburgh, 2017) and risk premia to claims of value-payouts being steeper with the horizon than those of growth-payouts (Hansen, Heaton, and Li, 2008; Giglio, Kelly, and Kozak, 2021, henceforth GKK21).

Notably, the equilibrium dynamics of our model are robust to the unconditional properties of the term structure of equity. Namely, two features of our model are noteworthy. First, the model mechanism driving the cyclicality of equity yields and term premia holds irrespective of the sign of their unconditional slope. Second, our model reconciles standard asset pricing moments with sizable short-term risk premia *independently* of the sign of the unconditional equity term premium—the main challenge posed by BBK12 to leading models. Thus, our economic mechanism is not affected by the empirical concern posed by Bansal, Miller, Song, and Yaron (2021) that short samples may lead to biased estimates of the unconditional slope by not properly capturing the alternation of good and bad economic conditions and the ensuing term premium.[2]

Our empirical analysis provides tight support to the model mechanism and its predictions. We estimate a simple measure of EGV from survey forecasts of economic growth, which is an observable and genuine measure of investors' expectations. We document that EGV rises during economic downturns, as in Bansal, Kiku, Shaliastovich, and Yaron (2014)—a stylized pattern that we feed into our model.

Then, we document four stylized facts that arise as endogenous outcomes in our equilibrium model, supporting its economic mechanism. First, as predicted by the model, we provide evidence that the market price-dividend ratio decreases with EGV, whereas its volatility increases with EGV. This result allows us to link macroeconomic fundamentals with the cyclicality of the equity term premium, estimated instead through the price-dividend ratio by Gormsen (2021). Second, the slope of the equity yields is strongly negatively correlated with

[2] We also show that the model mechanism and predictions are robust to a heteroscedastic specification of downward-sloping risk. Data suggests that a small fraction of transitory shocks' conditional variance comoves with our EGV. A model extension accomodates for such a pattern and still is able to explain the dynamics of the equity term structure.

EGV, consistently with the model and with the procyclical dynamics of the equity yields slope documented by van Binsbergen et al. (2013). Third, EGV predicts the realized slope of equity returns with a positive coefficient. Thus, EGV is a good candidate to drive the countercyclical dynamics of the equity term premium, in accord with the model and with the empirical findings of Gormsen (2021). Fourth, EGV predicts the value firms' return and the value-minus-growth return with a positive coefficient, whereas it does not predict the growth firms' return. Thus, EGV credibly drives the countercyclical dynamics of the value premium, consistent with our model. Moreover, EGV strongly predicts the value firms' return at long horizons, in accord with the model mechanism that produces upward-sloping compensations to the claims of value payouts—as documented by GKK21. Therefore, we empirically and theoretically connect the dynamics of both equity yields and equity term premia to macroeconomic fundamentals.[3]

Given our focus on the term structure of equity, our assumptions aim at correctly describing the timing of fundamentals' risk and how this transmits to the equilibrium state-price density. Peculiarly, our economy assumes consumption and payouts cointegration and limited market participation (Greenwald, Lettau, and Ludvigson, 2014; Marfè, 2017). These assumptions are empirically motivated and play a relevant role in shaping fundamentals' risk across the horizon. Cointegration implies that payout risk is downward-sloping with the horizon (Belo, Collin-Dufresne, and Goldstein, 2015; Marfè, 2016), whereas limited market participation implies that market participants' consumption is much more correlated with payouts than aggregate consumption (Berk and Walden, 2013).[4] Thus, by affecting the timing of fundamentals' risk, these assumptions can help understand the properties of the term structure of equity in equilibrium. Indeed, we verify that our model calibration matches

[3]The results of the empirical analysis are robust to many alternative specifications of EGV. Moreover, EGV subsumes alternative components of growth in explaining the slope of the equity term structure.

[4]Berk and Walden (2013) show that limited market participation arises endogenously because labor markets provide risk-sharing to workers. Consistently, a major fraction of workers does not invest in the financial markets and the consumption of market participants is more correlated with corporate payouts and equity returns than aggregate consumption (Mankiw and Zeldes, 1991; Guvenen, Schulhofer-Wohl, Song, and Yogo, 2017). However, market participants' consumption is subject to aggregate consumption long-run risk (Malloy, Moskowitz, and Vissing-Jørgensen, 2009), in accord with cointegration.

both the downward-slope of payout risk as well as the predictability of payout growth by the payout-to-consumption ratio across the horizons. Instead, many models disregard such empirical patterns and strongly overestimate payout risk at long horizons—further amplified under preferences for the early resolution of uncertainty. Moreover, we should avoid that the model explains sizable short-term risk premia because of a misspecified state-price density that weighs too much on short-term risk. Therefore, we exploit the state-price density decomposition of Alvarez and Jermann (2005) and Hansen and Scheinkman (2009): We verify that under our calibration, the fraction of state-price density volatility due to its permanent component does not violate the high lower bound estimated by Alvarez and Jermann (2005). This supports the way the timing of fundamentals' risk gets priced in equilibrium. Consistently, the model quantitatively endogenizes the decay rate of equity yields' sensitivity to EGV across the horizon, which generates the conditional dynamics of the equity term structure, that is the core of our analysis.

Several works study the term structure of equity in light of macroeconomic risk. Economic channels that have been investigated are beliefs formation (Croce, Lettau, and Ludvigson, 2015), financial leverage (Belo et al., 2015), disaster recovery (Hasler and Marfè, 2016), labor costs rigidity (Marfè, 2017), production with learning (Ai, Croce, Diercks, and Li, 2018), alternative preferences (Andries, Eisenbach, and Schmalz, 2019), irrational beliefs (Cassella, Golez, Gulen, and Kelly, 2022), and reinvestment risk (Gonçalves, 2021). We complement this literature by providing a parsimonious equilibrium framework that explains the rich *conditional* dynamics of equity slope. We share with Ai et al. (2018) the idea that EGV drives the equity term structure: We complement their evidence by using survey data and by documenting a robust link over a much longer sample (and more economic cycles), by exploiting the GKK21 synthetic equity yields.[5] Moreover, while they investigate a production economy with learning, we consider a more parsimonious framework and study

[5] Ulrich, Florig, and Seehuber (2022) make use of I/B/E/S survey data for firm dividend growth and build a model-free measure of the short-term dividend risk premium on the sample 2004-2021, which moves with economic conditions. Differently from our work they do not investigate the role of expected growth volatility.

limited market participation. While Ai et al. (2018) focus on the procyclical slope of equity yields, we also explain the more recent evidence about the countercylical dynamics of the equity term premium. Indeed, our paper then relates to Gormsen (2021), who proposes that two priced factors are needed to capture the negative average slope and the countercyclical variation of the equity term premium. Whereas his model is based on an exogenous state price density, our paper both *endogenizes* these dynamics in a general equilibrium setting and connects them to documented macroeconomic risk.

Our paper is also related to works about the cross-section of equity returns and the value premium. Among others, Berk, Green, and Naik (1999), Gomes, Yaron, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), and Zhang (2005) propose equilibrium models with heterogeneity either in growth options risk or in adjustment costs. We complement this literature by explaining the pricing of value and growth payouts across the horizon (GKK21)—further corroborating the model mechanism.[6]

The paper is organized as follows. The empirical analysis of Section II supports the main model assumption and predictions. Sections III, IV, and V describe the model and investigate its predictions. Section VI concludes. Proofs, further empirical tests, robustness exercises, and model extensions are in Appendices A, B, C, and D, respectively.

II Empirical Evidence

In this section, we first provide empirical support for the main model assumption about EGV and economic growth. Then, we generate a set of key stylized facts about the dynamics of the equity term structure and EGV, which can be jointly rationalized within our model.

[6] We complement Marfè (2015) and Ai et al. (2018), who also study the term structure of equity and the value premium through labor rigidity and investment, respectively. In a recent study, Hasler, Khapko, and Marfè (2020) show that rational learning helps understand the unconditional term structures of value and growth risk premia: we complement their approach and focus on dynamics.

Data. To build our baseline EGV measure, we obtain US mean growth forecasts from the Survey of Professional Forecasters (SPF) by the Federal Reserve Bank of Philadelphia about real gross domestic product (GDP). Survey data more genuinely capture market participants' actual expectations about the economy's fundamentals relative to inferring them from realized variables, and provide important insights into asset price dynamics (e.g., Barberis, Greenwood, Jin, and Shleifer, 2015; Greenwood and Shleifer, 2014). We lever on survey expectations to investigate the drivers of the equity term structure dynamics over the business cycle. Building on SPF data, we also construct alternative EGV measures from personal consumption expenditures (PCE), industrial production (IP), and corporate profits (CP) growth forecast, as well as a measure exploiting cross-sectional dispersion of GDP growth forecasts. The time series of SPF forecasts are the longest available, covering the period 1968-2019 at quarterly frequency, with the exception of PCE starting in 1981.

Information on actual macroeconomic conditions (e.g., realized growth rates, inflation, recessions) is from Federal Reserve Economic Data (FRED) by the Federal Reserve Bank of St. Louis. Information on aggregate stock market and equity term structure is from a variety of sources. As stock market index, we either rely on the value-weighted index from Center for Research in Security Prices (CRSP) or on the S&P 500 index from Robert Shiller's webpage. Monthly data on both indices are available throughout the period 1968-2019. We obtain information about equity yields at monthly frequency over different investment horizons from GKK21 and BBK12 for the period 1974-2019 and 1996-2009, respectively. Monthly data on value and growth portfolio value-weighted returns over 1968-2019 are from Kenneth French's website. All returns and monetary variables are expressed in real terms. Monthly observations are converted to quarterly frequency by summing them (for returns, which are logarithmic) or by taking the average (for other variables) over the quarter. Detailed information on data sources and main variables definitions is in Appendix Table C1.

Expected Growth Volatility (EGV). The main assumption of our model is that expected economic growth is heteroscedastic, so that its conditional volatility (EGV) is time-varying and, in particular, higher when the economic outlook deteriorates. To substantiate this assumption, we construct our baseline EGV measure by first filtering the conditional mean out of GDP growth forecasts $f_{t,t+1}$ with an AR(1) model and, then, by computing a moving average of the absolute residuals so obtained. We first estimate an AR(1) model:

$$f_{t,t+1} = \theta_0 + \theta_1 f_{t-1,t} + v_t. \tag{1}$$

Residuals from this regression, v_t, are not serially correlated, but their absolute values are. Thus, we build a simple measure of EGV (or conditional volatility):

$$\sigma_{t,t+1} = \frac{1}{n} \sum_{i=0}^{n-1} |v_{t-i}|. \tag{2}$$

A moving average of four lags produces a good fit of residuals. The inferred time series of conditional volatility $\sigma_{t,t+1}$ is our main EGV measure and explanatory variable.[7]

Importantly, EGV is negatively and significantly correlated with expected growth, at −18% (p-value of 0.012). When investors have low expectations about future growth, forecasts are more volatile. This evidence supports the assumption of our general equilibrium model that expected growth is decreasing in its conditional volatility. Another approach to detect the negative relation between EGV and expected growth is to observe the relation between EGV and economic variables that capture the economic outlook and are informative about economic prospects (e.g., Colacito and Croce, 2011). The negative correlation between EGV and expected growth suggests that EGV is a countercyclical measure of the state of the economy. We confirm this intuition in Table 1 through contemporaneous regressions of EGV on macroeconomic and financial measures, as we know from previous research that business and financial cycles interact (e.g., Adrian, Boyarchenko, and Giannone, 2019). EGV exhibits a positive and significant relation with an indicator for recessions as dated by the National

[7] Appendix Table C2 documents that the EGV specification of Eq. (2) provides a better fit than many alternatives, including (G)ARCH models as well as EGV measures built from IP, CP, and PCE growth forecasts instead of GDP ones. However, all these measures are highly correlated and we show below that our results are robust with respect to the EGV specification.

Table 1: The Cyclicality of EGV

	σ				
	(1)	(2)	(3)	(4)	(5)
Constant	0.219***	0.235***	-0.000	0.240***	1.132***
	(9.74)	(10.93)	(-0.01)	(12.99)	(6.04)
NBER Recession	0.162***				
	(4.46)				
Detrended Labor Share		1.583***			
		(3.45)			
Default Spread			0.226***		
			(5.70)		
NFCI				0.099***	
				(7.02)	
ln(P/D)					-0.245***
					(-4.85)
Observations	205	205	205	196	205
Adj. R^2	0.10	0.08	0.29	0.29	0.28

Note. This table reports estimates from contemporaneous regressions at quarterly frequency of EGV (σ) on selected measures of macroeconomic (an indicator for NBER recessions and the detrended labor share) and financial conditions (the default spread, the NFCI, and the logarithm of the price-dividend ratio of the CRSP value-weighted index) over the period 1968-2019. Coefficient estimates are multiplied by 100 to favor readability. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Bureau of Economic Research (NBER), the detrended labor share, the Chicago Fed's National Financial Conditions Index (NFCI)—capturing the tightness of financial conditions on capital markets—and the default spread. It also exhibits a negative and significant relation with the logarithm of the price-dividend ratio of the CRSP value-weighted index.

EGV and Expected Growth. We are now interested in linking our EGV measure to the short- and long-run components of economic growth. In particular, consider the following representation of growth rates:

$$g_{t+1} = \Delta\text{permanent}_{t+1} + \Delta\text{transitory}_{t+1}, \tag{3}$$

where the process $\text{permanent}_{t+1} \sim I(1)$ is integrated and captures the long-run trend, and the process $\text{transitory}_{t+1} \sim I(0)$ is stationary and induces transitory fluctuations. Similarly, growth rates can be written in terms of conditional expected growth and innovations:

$$g_{t+1} = \mathbb{E}_t[g_{t+1}] + \epsilon_{g,t+1} = f_{t,t+1} + \epsilon_{g,t+1}. \tag{4}$$

To make the decomposition in Eq. (4) operational, we use the SPF forecast $\epsilon_{g,t+1}$ as a proxy for conditional expected growth $f_{t,t+1}$, thus finding as the difference between the actual growth rate g_{t+1} and its forecast.

Building on the documented negative relation between EGV and expected growth, we estimate a linear regression:

$$f_{t,t+1} = a + b\,\sigma_{t,t+1} + \epsilon_{f,t} \quad \Rightarrow \quad \begin{cases} \hat{f}_{t,t+1} = \hat{a} + \hat{b}\,\sigma_{t,t+1}, \\ e_{f,t} = f_{t,t+1} - \hat{f}_{t,t+1}. \end{cases} \tag{5}$$

Parameter $\hat{b}$ is negative and significant. Then, actual growth rates can be rewritten as

$$g_{t+1} = \hat{f}_{t,t+1} + e_{f,t} + \epsilon_{g,t+1}. \tag{6}$$

This representation is useful to understand the link between EGV and the short- and long-run components of economic growth. Indeed, in Appendix B we find that

$$\sum\nolimits_t \hat{f}_{t,t+1} \sim I(1) \;\; \Rightarrow \;\; \Delta\text{permanent}_{t+1} \;\approx\; \hat{f}_{t,t+1}, \tag{7}$$

$$\sum\nolimits_t e_{f,t} + \sum\nolimits_t \epsilon_{g,t+1} \sim I(0) \;\; \Rightarrow \;\; \Delta\text{transitory}_{t+1} \;\approx\; e_{f,t} + \epsilon_{g,t+1}. \tag{8}$$

Notice that the process $\sum_t \hat{f}_{t,t+1}$ in Eq. (7) is integrated by construction. By contrast, the stationarity of the process Eq. (8) is less straightforward to determine. The term $\sum_t \epsilon_{g,t+1}$ alone is integrated by construction, whereas the cumulated sum of $e_{f,t} + \epsilon_{g,t+1}$ is stationary (Appendix Table B1). The interpretation is that EGV alone captures the long-run component of expected growth, whereas the residual $e_{f,t}$ from Eq. (5) captures its short-run component $\mathbb{E}_t[\Delta\text{transitory}_{t+1}]$:

$$\mathbb{E}_t[g_{t+1}] = f_{t,t+1} = \underbrace{f^{\text{long}}_{t,t+1}}_{\equiv \hat{f}_{t,t+1}} + \underbrace{f^{\text{short}}_{t,t+1}}_{\equiv e_{f,t}}. \tag{9}$$

EGV alone allows to identify the long-run component $f^{\text{long}}_{t,t+1}$ of expected growth: any potential missing term should be negligible, otherwise the process in Eq. (8) would not be stationary.

This interpretation is further supported by the fact that $\Delta\text{transitory}_{t+1}$, constructed as in Eq. (8), is strongly correlated (95.7%) with changes in output gap—a well known proxy for the stationary component of growth (e.g., Kamber, Morley, and Wong, 2018). Consistently, we also document that transitory_{t+1} is procyclical (Appendix Figure B1 and Table B2).

After establishing that EGV is an important component of long-run expected growth, we turn to conditional variation of the long- and short-run components of economic growth. Based on Eq. (7)-(8), the conditional volatility of the long-run component $\Delta\text{permanent}_{t+1}$ is captured by EGV. Consistently, we provide evidence pointing to quantitatively negligible heteroscedasticity of the short-run component $\Delta\text{transitory}_{t+1}$ (Appendix Figure B2).

We specify the assumptions of the general equilibrium model in Section III starting from the stylized facts presented so far. In summary, (i) economic growth is driven by a heteroscedastic integrated component (i.e., permanent_{t+1}) and a homoscedastic stationary component (i.e., transitory_{t+1}) and (ii) the long-run component of expected growth (i.e., $f^{\text{long}}_{t,t+1}$) is negatively related with the conditional variance of the integrated component (i.e., $\sigma_{t,t+1}$), that is our EGV measure. We next document four empirical facts concerning the relation between EGV and (i) prices and price volatility, (ii) the equity yield slope, (iii) the equity term premium, and (iv) dividends and returns of value and growth firms. All these patterns endogenously arise from our general equilibrium model.

EGV, Prices, and Price Volatility. Based on the evidence of Table 1, we examine the relation between EGV and the logarithm of the price-dividend ratio in more depth. Theoretical models predict that the price-dividend ratio is driven by the latent factors that affect the distribution of aggregate cash flows. We consider the following regression:

$$\ln(\mathrm{P}_t/\mathrm{D}_t) = \alpha + \beta\sigma_{t,t+1} + \varepsilon_t.$$

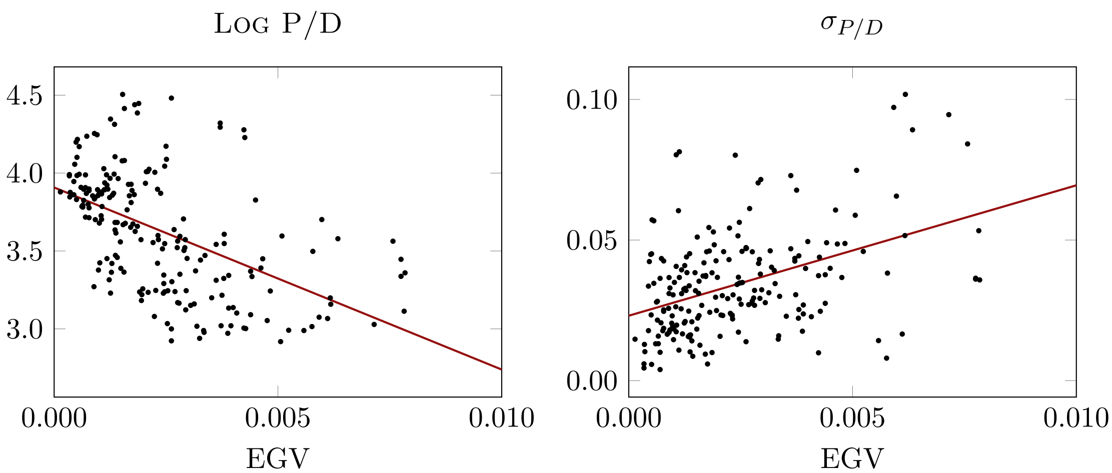

Figure 2: EGV and the Price-Dividend Ratio. This figure shows the scatter plots of either the log price-dividend ratio (left graph) or log price-dividend ratio volatility (right graph) of the CRSP value-weighted stock market index ($\sigma_{\mathrm{P/D}}$) against the EGV estimated from GDP growth forecasts (σ) for the period 1968-2019.

In the left panel of Figure 2, we observe that EGV decreases with the logarithm of the price-dividend ratio (correlation of -54%, statistically significant at the 1% level).[8] We then study the correlation between EGV and the conditional volatility of the logarithm of the price-dividend ratio, which we obtain following the same approach as in Eq. (2). In the right panel of Figure 2, we estimate the regression:

$$\sigma_{t,t+1}^{\ln(\mathrm{P/D})} = \alpha + \beta\sigma_{t,t+1} + \varepsilon_t.$$

We observe a positive correlation of 45% between EGV and the conditional volatility of the price-dividend ratio, significant at the 1% level. These results conform with the literature on macroeconomic volatility and uncertainty (Bansal et al., 2014; Boguth and Kuehn, 2013).

EGV and the Equity Yield Slope. The second empirical pattern regards the relation between EGV and the slope of equity yields, which is defined as the difference between the long- and the short-maturity equity yield at each point in time. van Binsbergen et al. (2013) illustrate that such a slope is procyclical: Short-maturity equity yields are lower than long-maturity ones during economic expansions, whereas they exceed long-maturity ones during

[8] This negative correlation can arise in a model where investors feature an elasticity of intertemporal substitution above one—i.e., the substitution effect dominates the wealth effect (Bansal and Yaron, 2004).

Table 2: Equity Yield Slope and EGV

Panel A

	Equity Yield Slope (GKK21)			
	(1) 10Y-2Y	(2) 25Y-2Y	(3) 100Y-2Y	(4) MKT-2Y
Constant	0.042***	0.068***	0.093***	0.065***
	(4.84)	(6.44)	(7.68)	(5.20)
σ	-10.062***	-15.556***	-20.303***	-21.809***
	(-3.50)	(-4.35)	(-4.88)	(-4.98)
Observations	180	180	180	180
Adj. R^2	0.15	0.20	0.24	0.25

Panel B

	Equity Yield Slope (BBK12)			
	(1) MKT-0.5Y	(2) MKT-1Y	(3) MKT-1.5Y	(4) MKT-2Y
Constant	0.141**	0.071**	0.045**	0.036**
	(2.45)	(2.34)	(2.09)	(2.12)
σ	-47.448***	-32.920***	-24.242***	-19.323***
	(-3.16)	(-4.14)	(-4.09)	(-4.19)
Observations	55	55	55	55
Adj. R^2	0.06	0.11	0.13	0.14

Note. This table reports estimates from contemporaneous regressions at quarterly frequency of the equity yield slope on EGV (σ). Panel A uses measures of the equity yield slope based on data by GKK21 for the period 1974-2019. Panel B uses measures of the equity yield slope based on data by BBK12 for the period 1996-2009. The maturities of the long and short legs considered to compute the equity yield slope are indicated at the top of each column. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

recessions. Bansal et al. (2021), Gormsen (2021) and GKK21 document similar patterns. We verify if the procyclical nature of the equity yield slope is—at least partially—channeled through EGV, which we have shown to be countercyclical.

Because of the hard-to-observe nature of the equity yield slope, we measure it in several ways. First, we use the model-implied equity yields made available by GKK21, which allow us to compute the equity slope at various maturities up to 100 years ($ey(t, \text{long})$) relative to the two-year yield ($ey(t, \text{short})$). For consistency with the second set of proxies based on BBK12 and described below, we also compute it using the dividend yield of the CRSP value-weighted index as $ey(t, \text{long})$. By spanning the period 1974-2019, the equity yields

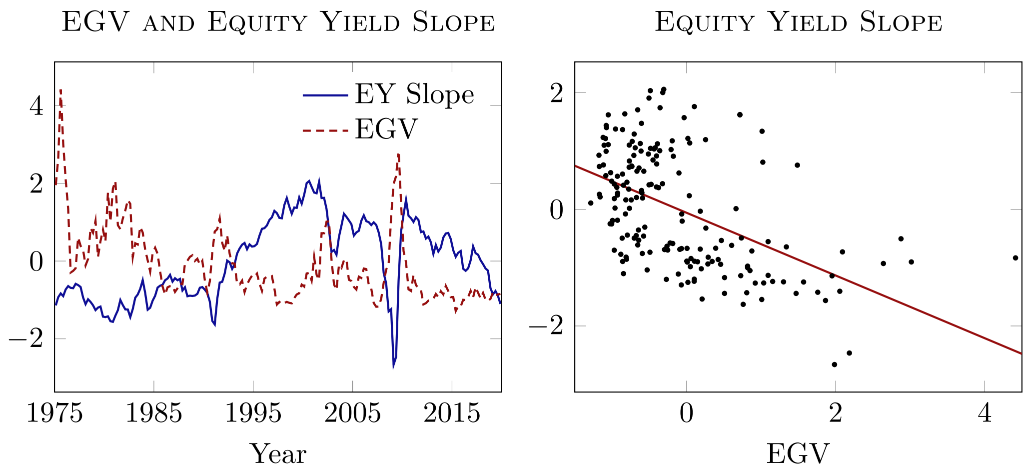

Figure 3: EGV and the Equity Yield Slope. The left of the figure shows the standardized time-series of the equity yield slope based (solid line) and the EGV (dashed line). The right panel shows the scatter plot of the standardized equity yield slope against the EGV. The equity yield slope is computed as the difference between the dividend yield of the CRSP value-weighted index and the model-implied two-year equity yield of GKK21 for the period 1974-2019.

by GKK21 are informative about the slope dynamics across different economic conditions. Second, we use data by BBK12, who extract information on short-maturity equity yields from option prices on the S&P 500 index. We then proxy for the slope by taking the difference between the S&P 500 dividend yield ($ey(t, \text{long})$) and short-maturity equity yields ($ey(t, \text{short})$), whose maturity ranges between 0.5 and two years.[9]

Table 2 reports the estimates from the regressions of these measures of the equity yield slope—based on data by GKK21 in Panel A, and by BBK12 in Panel B— on EGV:

$$ey(t, \text{long}) - ey(t, \text{short}) = \alpha + \beta\sigma_{t,t+1} + \varepsilon_t.$$

We observe a negative correlation between EGV and the slope of equity yields. The slope coefficients are negative and significant at the 1% confidence level across all the horizons. The R^2 from the regressions lies in the 6-25% range, pointing to a substantial explanatory ability of EGV as to the cyclical dynamics of the equity yield slope.[10]

[9]For both sets of proxies for the equity yield slope, results are not sensitive to using the S&P 500 or the CRSP value-weighted index dividend yield.

[10]To limit measurement error, we use spot equity yields ($ey(t, \tau)$) rather than forward ones ($fey(t, \tau)$) to compute the slope. Because $fey(t, \tau) = ey(t, \tau) - by(t, \tau)$, using forward yields would require subtracting risk-free yields ($by(t, \tau)$) of the appropriate maturity τ from each leg of the slope. This would be problematic

Consistently, the left panel of Figure 3, which focuses on a slope measure based on GKK21, documents a strong negative relation between EGV and the slope of equity yields. We observe a sharp mirror effect: the slope strongly decreases when EGV increases. The right panel of Figure 3 shows the corresponding scatter plot and linear fit. These results confirm that EGV is a major driver of the procyclical dynamics of the equity yield slope.[11] This evidence corroborates and extends the results in Ai et al. (2018): while they consider the equity yields in van Binsbergen et al. (2013) on the sample 2002-2010, we exploit the more recent GKK21's equity yields on the sample 1974-2020; moreover, we measure EGV from survey data about economic growth, whereas they build productivity volatility with a predictive approach.

EGV and the Equity Term Premium. Third, we study the relation between EGV and the equity term premium—that is, the compensation of long-term equity claims over the compensation of short-term equity claims. Gormsen (2021) finds that the equity term premium is time-varying and countercyclical. Namely, long-term equity premia are more sensitive to price levels than short-term equity premia. This implies that the equity term premium increases in bad times and decreases in good times. We test if EGV helps explain time-variation of the equity term premium in light of macroeconomic risk.

In the spirit of Gormsen (2021), we proxy for the equity term premium by taking the difference between the CRSP value-weighted index return and the return on the two-year dividend strip based on the corresponding equity yield by GKK21. Then, we compute the one- to ten-year ahead cumulative equity term premium. Finally, we perform regressions of

in our case, because we also use the 100-year equity yield and the market dividend yield as the long maturity leg, and it is not obvious to find information on risk-free rates for maturities above 30 years. Nonetheless, using spot equity yields from GKK21, in untabulated tests we compute the equity yield slope with forward yields for the 5-, 10-, and 20-year horizons (i.e., the maturities for which an appropriate risk-free rate is easily available). In each case, the correlation with the slope based on spot equity yields is above 99%.

[11] Appendix Table C3 further investigates the relation between the equity yield slope and expected growth. We estimate quantile regressions and document that the equity yield slope is informative about the tails of expected growth with more explanatory power for the left tail (Panel A). A very similar pattern is produced using EGV as explanatory variable (Panel B). These results corroborate the representation of Eq. (3)-(9), which motivates the assumptions of our equilibrium model in Section III.

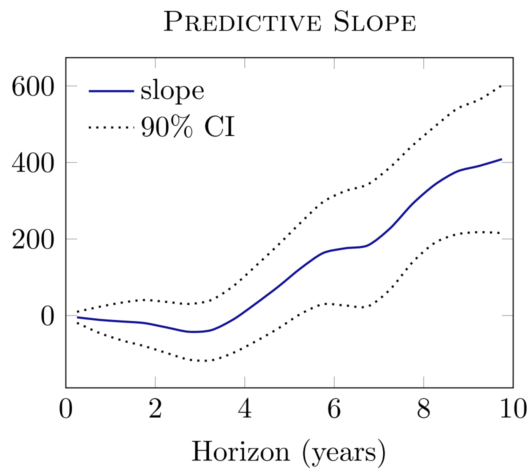

Figure 4: EGV and the Equity Term Premium. The figure shows the slope point estimates as well as 90% confidence intervals from predictive regressions of long-minus-short equity returns (equity term premium) on EGV across the horizons from one quarter up to ten years. In our measure of the equity term premium, the long leg is the CRSP value-weighted index return and the short one is the return on the two-year dividend strip obtained from model-implied yields by GKK21 for the period 1974-2019. The confidence intervals are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations.

the future cumulative equity term premium on the current EGV. Figure 4 points to a strong positive relation. The predictive slope for the current EGV is positive and statistically different from zero for predictive horizons beyond five years. This evidence suggests that EGV is a credible channel through which the equity term structure incorporates macroeconomic risk. Gormsen (2021) highlights the cyclicality of the equity term premium as measured by its correlation with the price-dividend ratio. We go a step further and show that EGV is a plausible link among the state of the economy, prices, and the term structure of equity. As we show below, this pattern is endogenized in our general equilibrium framework.

In Appendix Table C4, we use two different approaches—besides the baseline one—to measuring the equity term premium at the ten-year horizon. First, we resort to the GKK21 100-year strip return for the the long leg, keeping the two-year strip return as the short one. Second, we use the BBK12 short-term option-based strip returns to measure the short leg of the equity term premium (keeping the market return as the long one). In each case, we find evidence supportive of a positive and economically important relation between the equity

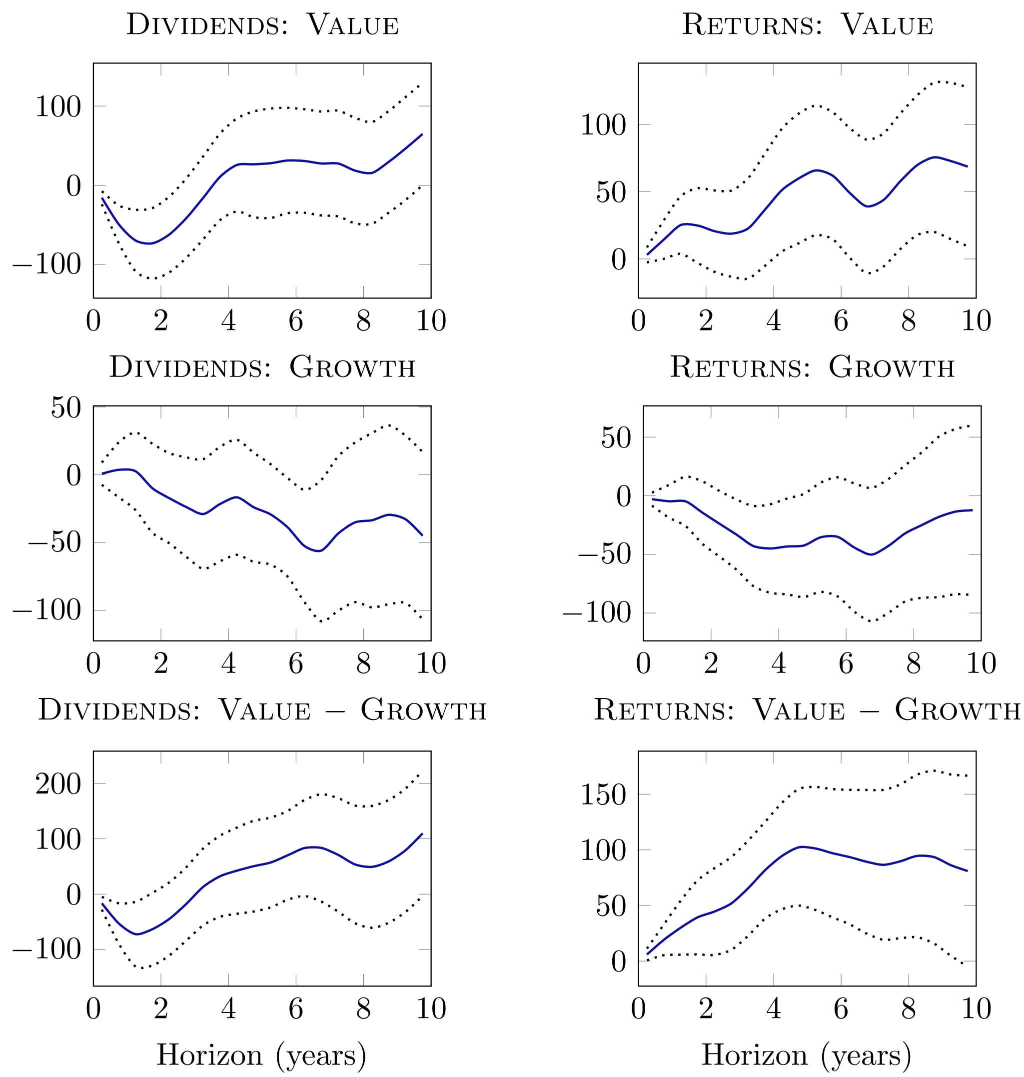

Figure 5: EGV and Value vs. Growth Firms. The figure shows the slope point estimates as well as 90% confidence intervals from the predictive regressions of dividend growth (left graphs) and returns (right graphs) for the value portfolio (top graphs), the growth portfolio (middle graphs), and the value-minus-growth portfolio (bottom graphs) on EGV across the horizons from one quarter up to ten years. The value (growth) portfolio corresponds to the top (bottom) decile of stocks sorted on the book-to-market ratio for the period 1968-2019. The confidence intervals are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations.

term premium and EGV, but the result is statistically insignificant at conventional levels when using strip returns by GKK21 alone (p-value of 0.101).

EGV and the Cross-Section of Returns. Fourth, we look at the relation between the EGV and the dividend growth rates as well as the returns of value firms vs. growth firms (Fama and French, 1992). In Figure 5, we estimate predictive regressions of cumulative dividend growth rates (left graphs) and returns (right graphs) from one quarter up to ten years ahead from either value firms, growth firms, or the value-minus-growth portfolio on EGV. We find a rather strong positive relation between EGV and both future dividend growth and future returns for value firms. The relation is instead negative but almost invariably insignificant for growth firms. EGV loads positively and in general significantly on future dividend growth and returns of the value-minus-growth portfolio.

All in all, we uncover that the value premium is related to the dynamics of the equity term structure through EGV. Value firms load more than growth firms on EGV. Since EGV drives long-term compensation, our results are consistent with the steeper term structure of value portfolios relative to growth portfolios estimated by GKK21. In turn, the value-minus-growth return is positively predicted by EGV and inherits its countercyclical behavior, in accord with the literature on the value premium (Guo et al., 2009).

The Cyclicality of the Term Structure of Equity. We now study the cyclical properties of the equity term structure dynamics in light of the recent findings in the literature (van Binsbergen et al., 2013; Bansal et al., 2021; Gormsen, 2021). In Appendix Table C5, we estimate univariate specifications of the equity yield slope and of the equity term premium on business-cycle proxies. Our estimates support the procyclicality of the equity yield slope, both when looking at business- and at financial-cycle measures. The picture becomes more nuanced for the equity term premium. It appears to correlate negatively with current macroeconomic conditions, but results are statistically insignificant at conventional levels. At the same time, the countercyclicality of the equity term premium stands out again when looking at financial-cycle variables.

We further explore the equity term structure dynamics in Appendix Table C6 by looking at the correlation of the equity yield slope and the equity term premium with EGV together with other possible drivers of growth, expected growth, and growth volatility (all orthogonalized with respect to EGV). Whereas only a few additional factors load significantly, our results point to EGV's *first-order* ability to satisfactorily capture the cyclicality of the equity yield slope and equity term premium. Thus, Appendix Table C6 highlights EGV as a key macroeconomic driver of the equity term structure and further corroborates the specification of Eq. (3)-(9), which we implement in the model of Section III.[12]

It is worth drawing a comparison between these findings and those in the literature. On the one hand, the EGV confirms the procyclical behavior of equity yields documented by van Binsbergen et al. (2013) and Bansal et al. (2021) by means of dividend strip prices, which are available only from the early 2000s. On the other hand, the lack of clear evidence with regards to the correlation of the equity term premium with "pure" macroeconomic variables, coupled with its positive (negative) correlation with EGV (price-dividend ratio), corroborates the analysis of Gormsen (2021) and highlights the role of EGV. Overall, the long time series of equity yields (1974:2019) from GKK21 points to the countercyclicality of the equity term premium, which incorporates macroeconomic risk as measured by EGV.

Alternative EGV Specifications. In Appendix Table C8, we test the robustness of our results to using nine alternative EGV measures. First, we look at four different AR(MA)(1,1)-(G)ARCH(1,1) specifications of EGV of GDP. Second, we rely on a *non-generated* measure, namely the cross-sectional dispersion of GDP growth forecasts. Third, we obtain the EGV of GDP growth forecasts computed after filtering out a short-term business cycle component by means of the (de-trended) labor share of the nonfinancial corporate sector. Finally, we build the EGVs of PCE, IP, and CP growth forecasts by applying the approach of Eq. (1)-(2).

Like the baseline EGV, all these measures are strongly countercyclical as highlighted by the negative and significant relation with the logarithm of the price-dividend ratio and the positive and significant relation with its volatility (columns 1 and 2). Similarly, columns 3

[12] We also investigate the relation between the term structure of equity, EGV, and the conditional covariance between innovations in expected growth (defined as the residuals $\hat{v}$ from Eq. (1)) and the corresponding forecast errors. A decomposition of such a covariance shows that it directly depends on the conditional standard deviation of $\hat{v}$, which is akin to our main EGV measure. We want to verify if also the other components of the covariance have explanatory power with respect to the term structure of equity. Appendix Table C7 shows that, generally, these components are neither statistically nor economically significant, reinforcing the interpretation of EGV as a crucial driver of equity term structure dynamics over the business cycle.

and 4 confirm that these alternative EGV measures capture the procyclicality of the equity yield slope too. At the same time, the positive relation of the different EGV measures with the equity term premium (column 5) remains economically and statistically significant. We also qualitatively reproduce the result about future dividend growth rates and returns of the value-minus-growth portfolio (columns 6 and 7). Although some of the alternative EGV measures have insignificant predictive power with respect to value-minus-growth returns, the sign and magnitude of the coefficients are largely unchanged.

Summary. Overall, the analysis of this section supports the idea that EGV is a major driver of the term structure of equity. When investors experience a higher conditional volatility of expected growth, (i) economic conditions deteriorate, (ii) prices decline, (iii) the slope of equity yields drops, (iv) the equity term premium increases, and (v) the slope of value claims becomes steeper relative to the slope of growth claims, leading to an increase in the value premium. The next section illustrates a parsimonious general equilibrium model that jointly endogenizes all these effects by building on the stylized evidence of Eq. (3)-(9).

III The Model

This section describes a general equilibrium model that captures the main properties of the equity term structure in light of macroeconomic risk, as documented in Section II.

The Economy. A representative firm produces a cash-flow stream, C, which constitutes the revenues from production distributed to workers and shareholders. Workers receive wages, W, and shareholders receive payouts, D, such that $C = W + D$. In the spirit of Berk and Walden (2013), we assume limited market participation.[13] That is, workers do not access

[13] Recent asset pricing models assuming limited market participation are Greenwald et al. (2014), Marfè (2017), and Lettau, Ludvigson, and Ma (2019). Although unnecessary for the qualitative predictions of the model about the equity term structure dynamics, the assumption of limited market participation helps generate sizable short-term risk premia. Moreover, it allows for tractability and comparability with endowment economy asset pricing models. Section V.C illustrates the alternative case of full market participation.

financial markets and consume their wages, whereas shareholders act as the representative agent in the stock market and consume payouts. Shareholders feature recursive preferences (Kreps and Porteus, 1979; Epstein and Zin, 1989; Weil, 1989; Duffie and Epstein, 1992):

$$U_t \equiv \left[(1-\beta^{dt})\hat{C}_t^{\frac{1-\gamma}{\theta}} + \beta^{dt}\mathbb{E}_t\left(U_{t+dt}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{10}$$

where $\hat{C}$ is a consumption process, β is the time discount factor, γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution, and we define $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Aggregate consumption dynamics depend on two components. The first is a permanent shock driven by time-varying expected growth. It gives rise to a stochastic trend and induces upward-sloping risk with the horizon—i.e., the variance of growth rates increases with the horizon. The second component is a transitory (short-term) shock z_t, which produces stationary fluctuations and induces downward-sloping risk with the horizon. The two shocks jointly allow for flexible term structures of risk. Consistent with the empirical evidence in Section II, the permanent shock is heteroscedastic, with a conditional variance negatively related with expected growth. Aggregate consumption dynamics follow:

$$d\log C_t = (\mu + \bar{x} - x_t)dt + dz_t, \tag{11}$$

where the permanent and the transitory components are governed by:

$$dx_t = \lambda_x(\bar{x} - x_t)dt + \sigma_x\sqrt{x_t}dB_{x,t}, \tag{12}$$

$$dz_t = -\lambda_z z_t dt + \sigma_z dB_{z,t}. \tag{13}$$

Brownian shocks $B_{x,t}$ and $B_{z,t}$ are independent, in line with the data (see Appendix B).[14]

Aggregate consumption, wages, and payouts are all subject to the permanent shock and, thus, are cointegrated (Lettau and Ludvigson, 2005). Moreover, the rigidity of labor costs (Menzio, 2005; Marfè, 2017) with respect to short-term fluctuations lead to income insurance from shareholders to workers, which induces a leverage effect on payouts. We parsimoniously

[14]The simple and tractable dynamics of Eq. (11)-(13) well capture the stylized evidence documented in Section II (see also Appendix B) and closely correspond to Eq. (3)-(9).

capture these effects by assuming that wages and payouts respectively satisfy:

$$W_t = \omega(z_t) C_t \qquad \text{and} \qquad D_t = (1 - \omega(z_t)) C_t,$$

where the wage share $\omega(z_t)$ is a function of the transitory (short-term) shock z_t:[15]

$$\omega(z_t) = 1 - \delta e^{(\phi - 1) z_t}.$$

Leverage $\phi \geq 1$ makes payouts (respectively, wages) more (less) exposed to short-term shocks than the firm's total cash flow. Consistently, payouts evolve as follows:

$$d \log D_t = (\mu + \bar{x} - x_t) dt + \phi \, dz_t. \tag{14}$$

Although parsimonious, these dynamics account for many empirical stylized facts, such as: (i) cointegration among consumption, wages, and payouts (Lettau and Ludvigson, 2005), (ii) excess volatility of payouts over consumption at short horizons (Belo et al., 2015), (iii) variance ratios of payout and wage growth rates lying respectively below and above those of consumption (Marfè, 2017), (iv) countercyclical wage share (Ríos-Rull and Santaeulàlia-Llopis, 2010), and (v) countercyclical expected growth volatility (see Section II).

The shock x_t—the key variable of the model—drives the conditional variance of growth:

$$\sigma_Y^2(t, \tau) = \frac{1}{\tau} \log \frac{\mathbb{E}_t[Y_{t+\tau}^2]}{\mathbb{E}_t[Y_{t+\tau}]^2} = s_{Y,0}(\tau) + s_{Y,x}(\tau) x_t, \qquad Y = \{C, D\},$$

where $s_{Y,x}(\tau) > 0$ increases with the horizon τ. Both x_t and z_t drive expected growth:

$$\mu_Y(t, \tau) = \frac{1}{\tau} \log \frac{\mathbb{E}_t[Y_{t+\tau}]}{\mathbb{E}_t[Y_t]} = m_{Y,0}(\tau) + m_{Y,x}(\tau) x_t + m_{Y,z}(\tau) z_t, \qquad Y = \{C, D\},$$

where $m_{Y,x}(\tau) < 0, m_{Y,z}(\tau) < 0$ decrease with the horizon τ. In turn, in line with the empirical evidence, the following relations hold:

$$\begin{aligned} -1 \; &< \operatorname{corr}(\sigma_Y^2(t,\tau), \, \mu_Y(t,\tau)) \; < \; 0, \qquad Y = \{C, D\}, \\ 0 \; &< \operatorname{corr}(\mu_C(t,\tau), \, \mu_D(t,\tau)) \; < \; 1. \end{aligned}$$

In the following, we show that the shock x_t—the model counterpart of EGV—is key to rationalize the main empirical properties of the equity term structure in general equilibrium.

[15]The function $\omega(z_t)$ belongs to $(0, 1)$ with probability very close to one because $\delta \approx 10\%$ is small in the data. A complementary channel for short-term levered payouts is sticky financial leverage (Belo et al., 2015).

State-Price Density and Equity Returns. Recursive preferences lead to a non-affine state-price density. To solve for prices and preserve tractability, we follow the methodology of Eraker and Shaliastovich (2008), which is based on the Campbell and Shiller (1988)'s log-linearization.[16] The continuous time (continuously compounded) log-return on equity—the claim on the shareholders' consumption D_t—follows

$$d\log R_t = k_0 dt + k_1 d(pd_t) - (1-k_1)pd_t dt + d\log D_t,$$

where $pd_t = \log(P_t/D_t)$ and the endogenous constants k_0 and k_1 satisfy

$$k_0 = -\log\left((1-k_1)^{1-k_1}k_1^{k_1}\right) \qquad \text{and} \qquad k_1 = e^{\mathbb{E}(pd_t)}/\left(1+e^{\mathbb{E}(pd_t)}\right).$$

The following Euler equation characterizes the equilibrium state-price density, M_t:

$$\mathbb{E}_t\left[\exp\left(\log\frac{M_{t+\tau}}{M_t} + \int_t^{t+\tau} d\log R_s\right)\right] = 1. \tag{15}$$

In turn, the state-price density satisfies

$$d\log M_t = \theta\log\beta dt - \frac{\theta}{\psi}d\log D_t - (1-\theta)d\log R_t. \tag{16}$$

To solve for the return on equity and, in turn, the state-price density, we conjecture that pd_t is affine in the vector of state variables. Then, the Euler equation is used to solve for the coefficients. In turn, the state-price density has dynamics:

$$\frac{dM_t}{M_t} = -r_t dt - \Omega_x(x_t)dB_{x,t} - \Omega_z dB_{z,t}. \tag{17}$$

In this equation, the risk-free rate is affine in the shocks x_t and z_t, as follows:

$$r_t = r_0 + r_x x_t + r_z z_t,$$

where the coefficients r_x and r_z satisfy:

$$r_x = -\gamma - \frac{A_x(\gamma\psi-1)(1-k_1(1-\lambda_x))}{\psi-1} - \frac{A_x^2k_1^2\sigma_x^2(\gamma\psi-1)^2}{2(\psi-1)^2} \quad \text{and} \quad r_z = -\frac{\lambda_z\phi}{\psi}.$$

Moreover, the two equilibrium prices of risk are given by

$$\Omega_x(x_t) = \sigma_x\sqrt{x_t}\,\frac{k_1A_x(\gamma-1/\psi)}{1-1/\psi} \quad \text{and} \quad \Omega_z = \sigma_z\,\frac{k_1A_z(\gamma-1/\psi)}{1-1/\psi} + \sigma_z\,\gamma\phi,$$

[16] Campbell, Lo, and MacKinlay (1997), Bansal, Kiku, and Yaron (2012), and Hasler and Marfè (2016) show the high accuracy of the return log-linearization, which we assume exact hereafter.

where the price elasticities A_x and A_z are defined below. The equity price is given by

$$P_t = \int_0^\infty \mathbb{E}_t \left[\frac{M_{t+\tau}}{M_t} D_{t+\tau} \right] d\tau = D_t \exp\left(A_0 + A_x x_t + A_z z_t\right), \tag{18}$$

where

$$A_x = -\frac{2(1-1/\psi)}{1-k_1(1-\lambda_x)+\Xi} \quad \text{and} \quad A_z = -\frac{\phi\lambda_z(1-1/\psi)}{1-k_1(1-\lambda_z)},$$

with $\Xi = \sqrt{1-k_1\left(2(1-\lambda_x)+k_1(2(\gamma-1)\sigma_x^2-(1-\lambda_x)^2)\right)}$. Under plausible assumptions about preferences ($\gamma > 1/\psi, \psi > 1$), we have that the market prices of permanent and transitory risk satisfy $\Omega_x < 0$, $\Omega_z < 0, A_x < 0$, and $-\phi < A_z < 0$. Thus, prices relative to payouts decrease with both expected growth volatility and short-term shocks.

An application of Itô's Lemma provides the return variance:

$$\sigma_R^2(x_t) = x_t\sigma_x^2 A_x^2 + \sigma_z^2(\phi + A_z)^2,$$

and the equity premium:

$$RP(x_t) = x_t\sigma_x^2\left(\frac{\gamma-1/\psi}{1-1/\psi}A_x^2 k_1\right) + \sigma_z^2\left(\phi + A_z\right)\left(\gamma\phi + \frac{\gamma-1/\psi}{1-1/\psi}A_z k_1\right).$$

Under plausible preferences, the equity premium increases with expected growth volatility, as in long-run risk models (Bansal and Yaron, 2004), and the return variance moves negatively with prices, in accord with the volatility feedback (Campbell and Hentschel, 1992).

Term Structures of Equity and Bond. The price of the dividend strip with maturity τ has exponential affine solution:

$$P_{t,\tau} = \mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t}D_{t+\tau}\right] = D_t \exp\left(a_0(\tau) + a_x(\tau)x_t + (a_z(\tau)-\phi)z_t\right). \tag{19}$$

The deterministic functions $a_0(\tau), a_x(\tau)$, and $a_z(\tau)$ solve a system of ordinary differential equations. Closed-forms for the price elasticities $a_x(\tau)$ and $a_z(\tau)$ are in Appendix A.

Using the dividend strip price, we can compute the term structure of the dividend strip risk premium, $RP_{DS}(x_t,\tau)$, which is a function of only x_t and the maturity τ:

$$RP_{DS}(x_t,\tau) = x_t\sigma_x^2 a_x(\tau)\left(\frac{k_1 A_x(\gamma-1/\psi)}{1-1/\psi}\right) + \sigma_z^2 a_z(\tau)\left(\frac{k_1 A_z(\gamma-1/\psi)}{1-1/\psi} + \gamma\phi\right). \tag{20}$$

The dividend strip risk premium increases with expected growth volatility x_t. For $\gamma > \psi > 1$, the first and the second term of Eq. (20) respectively imply that the permanent shock (and, thus, expected growth volatility) induces an upward-sloping effect and the short-term shocks induce a downward-sloping effect on the term structure. When x_t is large, the upward sloping effect dominates and the price-payout ratio declines. When x_t is small, the downward-sloping effect dominates and the price-payout ratio rises. In turn, the equity term premium—that is, the slope of the dividend strip risk premium—is countercyclical as in Gormsen (2021).

Similarly, we compute the term structure of the forward equity yields. The forward equity yield with maturity τ is the difference between the equity yield and the riskless bond yield:

$$fey(t,\tau) = ey(t,\tau) - by(t,\tau), \tag{21}$$

where

$$ey(t,\tau) = -\frac{1}{\tau}\log(P(t,\tau)/D_t), \quad \text{and} \quad by(t,\tau) = -\frac{1}{\tau}\log B(t,\tau).$$

The price of the riskless bond with maturity τ is given by

$$B(t,\tau) = \mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t}\right] = \exp\left(b_0(\tau) + b_x(\tau)x_t + b_z(\tau)z_t\right). \tag{22}$$

The deterministic functions $b_0(\tau)$, $b_x(\tau)$, and $b_z(\tau)$ solve a system of ordinary differential equations. Closed-forms for the price elasticities $b_x(\tau)$ and $b_z(\tau)$ are in Appendix A.

Cross-Sectional Equity Returns. We introduce a cross-section of payout streams, to be interpreted as the payout of either firms or portfolios of stocks. Specifically, the cross-sectional payout, D_t^φ, has dynamics:

$$d\log D_t^\varphi = d\log D_t + \varphi(\bar{x} - x_t)dt + \sigma_\varphi dB_{\varphi,t}. \tag{23}$$

The loading φ captures the heterogeneous additional exposure to x_t in the cross-section.[17] Furthermore, the volatility parameter $\sigma_\varphi \neq 0$ allows for idiosyncratic risk. Following Eraker and Shaliastovich (2008), the log return on the stock paying out D_t^φ evolves as

$$d\log R_t^\varphi = k_0^\varphi dt + k_1^\varphi d(pd_t^\varphi) - (1-k_1^\varphi)pd_t^\varphi dt + d\log D_t^\varphi,$$

[17] For the sake of simplicity and exposition, we do not assume other forms of heterogeneity.

where k_0^φ and k_1^φ are endogenous constants. The price of this stock then can be written as

$$P_t^\varphi = \int_0^\infty \mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t} D_{t+\tau}^\varphi\right] d\tau = D_t^\varphi \exp\left(A_0^\varphi + A_x^\varphi x_t + A_z^\varphi z_t\right), \tag{24}$$

where coefficients A_0^φ, A_x^φ, and A_z^φ are derived in Appendix A. The price elasticity A_x^φ with respect to x_t is larger (smaller) in magnitude than the market price elasticity A_x if φ is larger (smaller) than zero. Thus, the larger the payout loading on expected growth volatility, the more pro-cyclical the valuation ratio and, hence, the more counter-cyclical the risk premium. Applying Itô's Lemma to Eq. (24), the corresponding risk premium is given by

$$RP^\varphi(x_t) = x_t \sigma_x^2 A_x^\varphi \left(\frac{k_1 A_x(\gamma - 1/\psi)}{1 - 1/\psi}\right) + \sigma_z^2 (A_z^\varphi + \phi)\left(\frac{k_1 A_z(\gamma - 1/\psi)}{1 - 1/\psi} + \gamma\phi\right). \tag{25}$$

The risk premium is increasing in the conditional volatility of expected growth.

Consider the payout streams associated with two loadings $\varphi_V > \varphi_G$. The valuation ratio associated to the payout with the higher exposure to x_t (i.e., φ_V) is lower and more cyclical than the valuation ratio associated to the payout with the lower exposure (i.e., φ_G). Thus, for $\varphi_V \gg \varphi_G$, we can interpret the former and the latter as the payout streams of value and growth firms, respectively. Therefore, the model-implied value premium is given by

$$VP(x_t) = RP^{\varphi_V}(x_t) \; - \; RP^{\varphi_G}(x_t) > 0.$$

Since the coefficient of x_t in Eq. (25) is positive and increasing in φ, then the value premium is positive and counter-cyclical, in accord with the empirical evidence. In particular, x_t is a driver of the value premium consistently with the predictive regressions in Section II.

To better connect this prediction with the model predictions about the term structure of equity, we look at the term structures of risk premia in the cross-section. The price of the claim that pays out $D_{t+\tau}^\varphi$ at maturity τ is given by

$$P_{t,\tau}^\varphi = \mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t} D_{t+\tau}^\varphi\right] = D_t^\varphi \exp\left(a_0^\varphi(\tau) + a_x^\varphi(\tau) x_t + (a_z^\varphi(\tau) - \phi) z_t\right). \tag{26}$$

The deterministic functions $a_0^\varphi(\tau)$, $a_x^\varphi(\tau)$, and $a_z^\varphi(\tau)$ solve a system of ordinary differential equations. Closed-forms for the price elasticities $a_x^\varphi(\tau)$ and $a_z^\varphi(\tau)$ are in Appendix A.

Applying Itô's Lemma to this strip price, we compute the term structure of the strip risk premium, which depends on x_t and the maturity:

$$RP^{\varphi}_{DS}(x_t,\tau) = x_t\sigma_x^2 a_x^{\varphi}(\tau)\left(\frac{k_1A_x(\gamma-1/\psi)}{1-1/\psi}\right) + \sigma_z^2 a_z^{\varphi}(\tau)\left(\frac{k_1A_z(\gamma-1/\psi)}{1-1/\psi}+\gamma\phi\right).$$

The strip risk premium increases with expected growth volatility. The larger the payout loading φ on x_t, the steeper the strip risk premium. In turn, the unconditional slope of the strip risk premium is larger for value firms than growth firms, as in GKK21.

IV Model Analysis

The next proposition summarizes the main model predictions, which are consistent with the empirical analysis of Section II. We study the dynamics of several equilibrium outcomes as a function of x_t, which is the model counterpart of EGV.

Proposition. *Under plausible parameters, the model predicts that:*

1. *The price-payout ratio decreases with EGV:* $\frac{\partial}{\partial x}\log P_t/D_t < 0$.
2. *The slope of the equity yields decreases with EGV:* $\frac{\partial^2}{\partial x\partial\tau} ey(t,\tau) < 0$.
3. *The slope of the dividend strip risk premium increases with EGV:* $\frac{\partial^2}{\partial x\partial\tau} RP_{DS}(x_t,\tau) > 0$.
4. *The value premium increases with EGV:* $\frac{\partial}{\partial x}\left(RP^{\varphi_V}(x_t) - RP^{\varphi_G}(x_t)\right) > 0$.

In the following, we analyze these predictions in detail. We present the model calibration and the predictions about standard moments and price dynamics. Then, we discuss the term structure of equity and its dynamics, which is our main focus. We also explore the predictions about cross-sectional returns and the value premium. Finally, we show that the dynamics of equity slope hold irrespective of the sign of the unconditional average and the degree of market participation.[18]

IV.A Calibration and Standard Moments

Table 3 provides our baseline setting. Economic fundamentals are described by long-run growth (μ), expected growth volatility ($\bar{x}, \sigma_x$, and λ_x), short-run shocks (σ_z and λ_z), and the

[18] We also illustrate a model extension with heteroscedastic transitory risk, which preserves our results.

Table 3: Model Parameters

Fundamentals	Symbol	Value
Long-run expected growth	μ	0.025
Expected growth volatility		
shift	$\bar{x}$	0.08
scale	σ_x	0.04
reversion	λ_x	0.15
Short-run shock		
scale	σ_z	0.025
reversion	λ_z	0.15
Payout leverage	ϕ	7.5
Preferences	Symbol	Value
Time discount factor	β	0.96
Risk aversion	γ	7.5
Elasticity of intertemporal substitution	ψ	1.5

leverage effect on payouts (ϕ). Parameters are set to match a wide array of moments from the time-series of fundamentals and financial returns. We assume standard values for the time discount factor (β), risk aversion (γ), and elasticity of intertemporal substitution (ψ).

Table 4 displays moments about consumption, payout, and financial returns from both the data and the model. The model is simulated at monthly frequency, and each simulation covers a time period comparable to the postwar experience. Simulated time-series are then aggregated at yearly frequency and, for each moment, we report selected percentiles.

Notably, the model matches quite well standard moments considered in the literature. Consumption and corporate payouts are cointegrated and, thus, their growth rates have similar sample averages (about 2.5%) close to their empirical counterparts. Consumption growth volatility is modest (about 3% vs. 1.8% in the data). The model captures the 1-year excess volatility of payout growth (about 18% vs. 15% in the data),[19] and the decline in payout growth volatility at long horizons (20-year volatility is about 10.5% vs. 8.5% in the data). Cointegration implies the stationarity of the payout-to-consumption ratio, whose

[19] In addition to corporate profits, Table 4 also reports an alternative measure of shareholders remuneration, dividends plus net repurchases, featuring a larger volatility (Belo et al., 2015). These two measures can be viewed as bounds, with respect to which the model performs well (see also Figure 6).

Table 4: Standard Moments

Moment	Data	Model 2.5%	Model 5%	Model 50%	Model 95%	Model 97.5%
Avg. consumption growth	0.021	0.007	0.010	0.025	0.038	0.040
Std. consumption growth	0.018	0.024	0.025	0.030	0.036	0.037
Avg. payout growth	0.030	0.005	0.008	0.025	0.041	0.043
Std. payout growth						
- *Corporate profits*	0.148	0.152	0.157	0.182	0.208	0.212
- *Dividends plus net repurchases*	0.266					
20-year std. payout growth						
- *Corporate profits*	0.083	0.019	0.028	0.105	0.222	0.249
- *Dividends plus net repurchases*	0.118					
Std. log payout-consumption ratio	0.228	0.177	0.186	0.255	0.355	0.378
Corr. consumption and payout exp. growth	0.822	0.244	0.316	0.663	0.849	0.874
Corr. payout exp. growth and exp. growth volatility	-0.331	-0.748	-0.701	-0.383	0.068	0.155
Avg. risk-free rate	0.007	-0.022	-0.017	0.008	0.032	0.037
Std. risk-free rate	0.025	0.021	0.022	0.031	0.045	0.048
Avg. excess equity return	0.068	0.025	0.031	0.057	0.085	0.090
Std. excess equity return	0.175	0.116	0.120	0.139	0.159	0.163
Avg. Sharpe ratio	0.388	0.178	0.216	0.409	0.625	0.666
Avg. log price-dividend ratio	3.435	3.121	3.135	3.204	3.271	3.284
Std. log price-dividend ratio	0.443	0.061	0.065	0.090	0.125	0.134
Avg. excess high-minus-low return	0.035	0.001	0.007	0.035	0.065	0.072
Std. excess high-minus-low return	0.129	0.106	0.109	0.126	0.144	0.147

Note. This table reports moment statistics from both data and model simulations. Model-implied statistics are moment quantiles from short-sample (75 years) simulations. The model is simulated at monthly frequency. Statistics are yearly moments if not stated otherwise. Consumption and payout (corporate profits) data are from the National Income and Product Accounts (NIPA). Returns are from K. French webpage. The price-dividend ratio is from R. Shiller webpage. Dividends plus net repurchases are from Belo et al. (2015).

volatility is captured quite well by the model (about 25% vs. 23% in the data). The model also captures the strongly positive correlation between expected growth of consumption and payouts (about 66% vs. 82% in the data) and the moderately negative correlation bewteen expected growth volatility and payout expected growth (about -38% vs. -33% in the data).

Consistent with the data, the risk-free rate implied by the model is low (about 0.8% vs. 0.7% in the data) and smooth (about 3% volatility vs. 2.5% in the data). Moreover, the model predicts a sizable equity premium that compares well with its empirical estimates (about 6% vs. 7% in the data). The excess return volatility in the model is somewhat smaller than in the actual data (about 14% vs. 17.5% in the data). However, the model Sharpe ratio is quite in line with the data (about 41% vs. 39% in the data). The model predicts a realistic value for the average log price-payout ratio (3.2 vs. 3.4 in the data) but its volatility is smaller than in the data (9% vs. 44% in the data). The model also generates a positive and sizable value premium that compares well with the historical average return on the high-minus-low (HML) portfolio (3.5% in both the model and the data) and its volatility (12.6% vs. 12.9% in the data).[20] Overall, whereas our focus is the term structure of equity and its dynamics, Table 4 suggests that the model performs well in describing the main properties of financial markets—then proposing a potential solution to the challenge posed by BBK12 to leading models in the literature.

The quantitative analysis of the model predictions, especially those regarding the term structure of equity, crucially depends on the balance between long-term shocks and short-term shocks. We devise three exercises to verify that this balance and its equilibrium implications are consistent with the data.

First, we compare the model term structure of payout volatility with that in the data. The left panel of Figure 6 displays the model-implied term structure up to 20 years. Volatility monotonically decreases from about 20% to 10%. The plot also shows the empirical volatility computed from either corporate profits or dividend plus net repurchases. The former decreases from about 14.8% to 8.3%, and the latter decreases from about 26.6% to 11.8%. Thus, the model volatility matches well the level and the timing of fundamental risk.

Second, we exploit cointegration and consider the model predictability of payout growth by the logarithm of the payout-consumption ratio:

$$(\log D_{t+\tau} - \log D_t)/\tau = \alpha + \beta \log D_t/C_t + \varepsilon_t, \qquad \tau \in (0.25, 10).$$

The middle panel of Figure 6 shows the median as well as the 2.5% and 97.5% percentiles

[20] We comment in Section IV.C about the setting of cross-sectional heterogeneity.

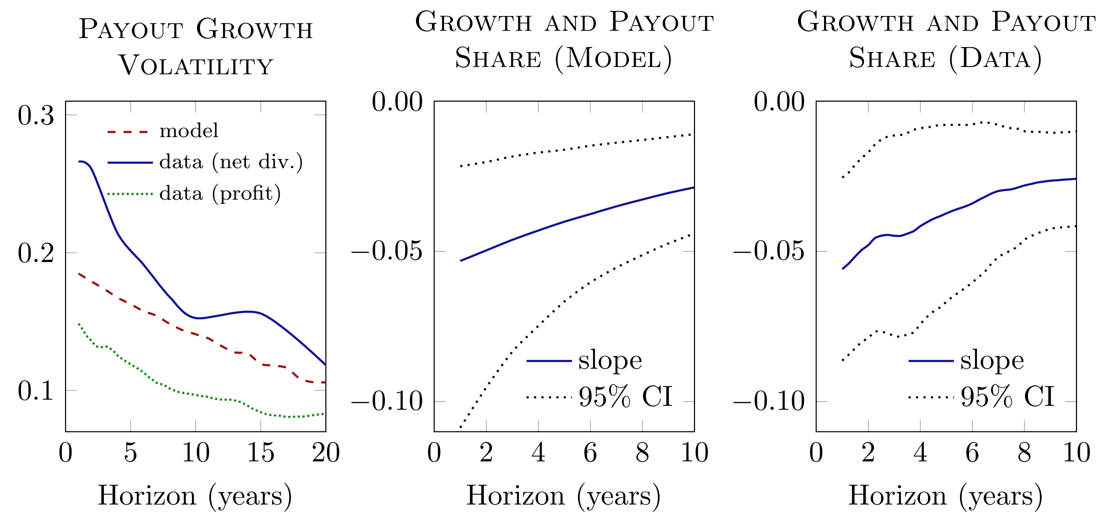

Figure 6: Cointegration and the Timing of Payout Risk. The left graph displays payout growth volatility as a function of the horizon in the model and in the data (corporate profits and net dividends). The middle and the right graph show the predictive coefficient from the regression of cumulative payout growth on the payout to consumption ratio across the horizon in the model and in the data, respectively.

of the predictive slope at any horizon between 1-quarter and 10 years. Slopes are negative and significant. This pattern arises because the model assumes that the ratio is positively driven by the stationary short-run shock z_t of consumption and payout. The right panel of Figure 6 displays the same regression estimates from actual data. Slopes are negative, and the Hansen-Hodrick 95% confidence interval documents that they are significant. The model well matches the sign and magnitude across the horizons of payout growth predictability.

Third, we exploit the lower bound on the fraction of state-price density volatility due to its permament component, introduced by Alvarez and Jermann (2005). The empirical estimates of the bound are close to and bounded above by one. Following Hansen and Scheinkman (2009), we compute the volatility ratio in our model and find values between 1.03 and 1.05 depending on the horizon. Thus, the bound is satisfied. The way our model transmits permanent and transitory risks from fundamentals to the endogenous state-price density is consistent with the empirical evidence: Sizable short-term compensations are not due to overweighing short-term risk.

Overall, the model captures well the timing of fundamental risk and how it gets priced in

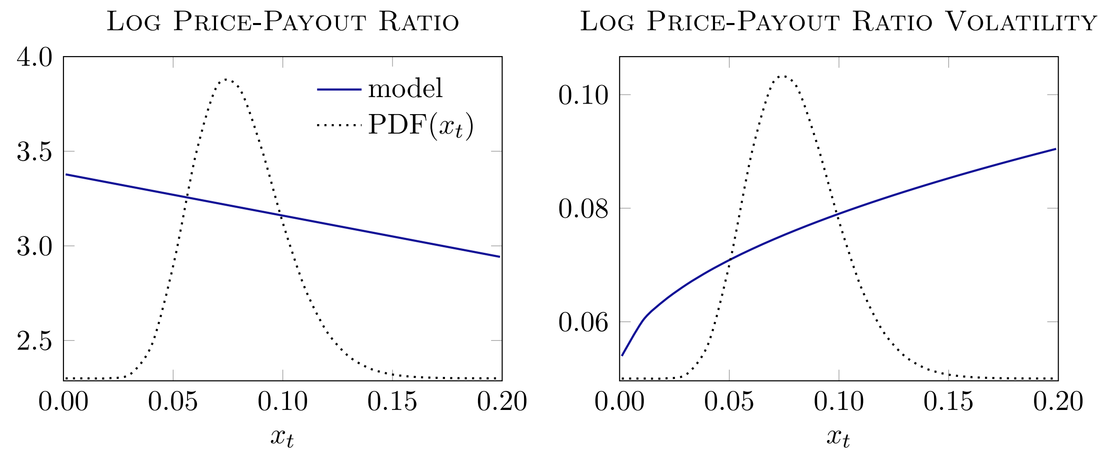

Figure 7: Price Dynamics. The figure reports the model log price to payout ratio (left graph) and its instantaneous volatility (right graph) as a function of EGV (x_t). The (standardized) unconditional density of EGV is superimposed.

equilibrium. Therefore, we believe that, although simple and parsimonious, our model represents a well-suited laboratory to understand the term structure of equity in equilibrium.[21]

Finally, Figure 7 shows that prices decline when EGV (x_t) increases, whereas their volatility rises. These model results (claim *1* of our Proposition) are consistent with the empirical evidence in Figure 2. In the following, we study the dynamics of the equity slope and refer to countercyclical behavior in terms of either a positive correlation with EGV or, equivalently, a negative correlation with the price-payout ratio—endogenizing Gormsen (2021)'s findings.

IV.B The Term Structure of Equity

The equity term structure and its dynamics are the key focus of our paper. We analyze the slope dynamics of equity yields and dividend strip risk premia (claims *2* and *3* of our Proposition), which have attracted substantial attention among researchers (BBK12, van Binsbergen et al. (2013), Gormsen (2021)). Our framework jointly rationalizes these empirical patterns in light of documented macroeconomic risk.

[21]In contrast, many models in the literature disregard cointegration and markedly overestimate long-horizon payout risk. Such a bias becomes even more relevant in combination with the preferences for the early resolution of uncertainty, which amplify the impact of long-horizon payout risk on asset prices. In Section V.C, we also discuss the assumption of limited market participation and its qualitative and quantitative impact on the model predictions.

Before investigating in detail the model predictions about the dynamics of the equity term structure and inspecting the model mechanism, we provide a quantitative assessment that the model-implied equity yields react to EGV, across the entire term structure, in a similar fashion to what we observe in actual data. Indeed, this is the key relation in this research. To this end, we estimate the loadings of EGV on the GKK21's equity yields across the maturity τ, which ranges from one to 100 years:

$$ey(t,\tau) = a_\tau + b_\tau \sigma_{t,t+1} + e_{\tau,t}, \qquad \forall \tau.$$

Coefficients b_τ are positive and statistically significant at all maturities. In order to generate an equity yield slope that decreases with EGV—a key stylized fact documented in Section II—we need that the loading b_τ decreases with the maturity τ. Actually, we observe that the long-maturity loading is about one order of magnitude smaller than the short-maturity loading.

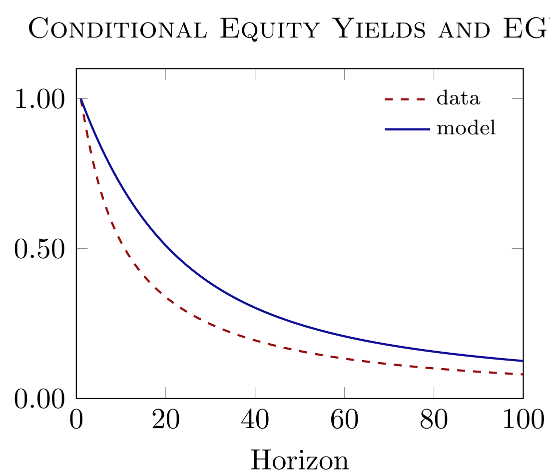

Figure 8: Conditional Equity Yields. The figure reports the slopes of univariate regressions of the GKK21's equity yield on EGV, standardized for the short-maturity level, as a function of the maturity and their model-implied counterparts.

We also build the model counterpart of the loadings b_τ. Using Eq. (19), the loading for maturity τ is simply given by

$$b_\tau^{\text{model}} = -\frac{1}{\tau} a_x(\tau) > 0, \qquad \forall \tau.$$

We then compare the loadings estimated from the data and the model-implied ones in Figure

8. The figure plots the loadings, standardized for the short-maturity level, as a function of the maturity. In this way, we can compare the decay rate of the equity yields sensitivity to EGV across the entire term structure. We observe that both the loadings estimated from the data and the model-implied ones provide a very similar shape, decreasing with the maturity. In particular, both quantities converge to a long-maturity level that is about 10% of the short-maturity one. Thus, the model captures well the heterogeneous response of the equity yields to EGV across the maturity, leading to a meaningful measure of the conditional equity yield slope—the core of our analysis.

Note that our calibration does not target any of the coefficients in Figure 8. The result is instead an endogenous equilibrium outcome and a consequence of a realistic weighting of short-term and long-term risks that we have set referring only to macroeconomic fundamentals, as documented in Figure 6 and Table 4. Armed with this result, we are going to investigate in detail the model-implied dynamics of the equity term structure and their underlying model mechanism.

In our model, permanent shocks induce upward-sloping risk, which inherits the time-varying and countercyclical properties of EGV. Conversely, transitory (short-term) shocks induce constant downward-sloping risk, because they are homoscedastic. As a result, the equity term premium is positive in bad times (in which EGV is high) and negative in good times (in which EGV is low)—i.e., the countercyclical dynamics of the equity term premium depend on the heteroscedastic nature of the main source of risk affecting long-term payouts.

The upper panels of Figure 9 display the forward equity yields and the dividend strip risk premia in expansions, recessions, and in the steady state. In the steady state ($x_t = \bar{x}$), forward equity yields are about flat across the horizon. When economic conditions deteriorate (EGV rises), short-term forward equity yields rise more than long-term ones. As a result, the slope of forward equity yields becomes negative in recessions. Conversely, when economic conditions improve (EGV falls), short-term forward equity yields decrease (and become negative) more than long-term ones. Thus, the slope becomes positive during expansions.

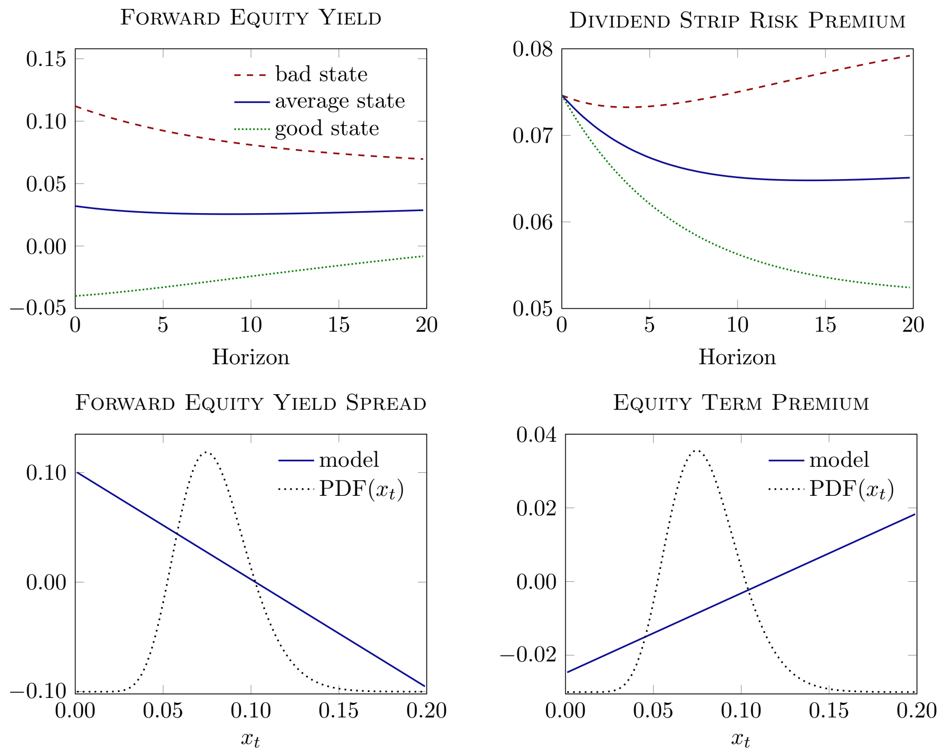

Figure 9: Equity Slope Dynamics. The upper panels of the figure report the model forward equity yield (left) and the model dividend strip risk premium (right) as a function of the horizon for several values of EGV (x_t). The lower panels of the figure show the model forward equity yield spread (left) and the model equity term premium (right) as a function of EGV (x_t). The (standardized) unconditional density of EGV (x_t) is superimposed.

Overall, the slope of forward equity yields is procyclical, as in van Binsbergen et al. (2013).

Consider now the dividend-strip risk premia. In the steady-state ($x_t = \bar{x}$), the risk premium is slightly downward-sloping (BBK12). When economic conditions deteriorate (EGV rises), long-term risk premia rise more than short-term ones. In turn, the slope of risk premia is positive during recessions. Conversely, when economic conditions improve (EGV falls), long-term risk premia decrease more than short-term ones. Thus, the slope is negative during booms. Overall, the equity term premium dynamics are countercyclical as in Gormsen (2021). The slope dynamics of the forward equity yield and the dividend strip premium can also be inspected through their infinite-horizon spreads:

$$\lim_{\tau\to\infty} fey(t,\tau) - \lim_{\tau\to 0} fey(t,\tau) \quad \text{and} \quad \lim_{\tau\to\infty} RP_{DS}(x_t,\tau) - \lim_{\tau\to 0} RP_{DS}(x_t,\tau).$$

The lower panels of Figure 9 report these spreads. The forward equity yield spread is decreasing with EGV, positive in expansions, and negative in recessions. Conversely, the spread of the dividend strip risk premium is increasing with EGV, negative in expansions and positive in recessions. Thus, the forward equity yield and dividend strip risk premium spreads move procyclically and countercyclically respectively and switch sign over time.

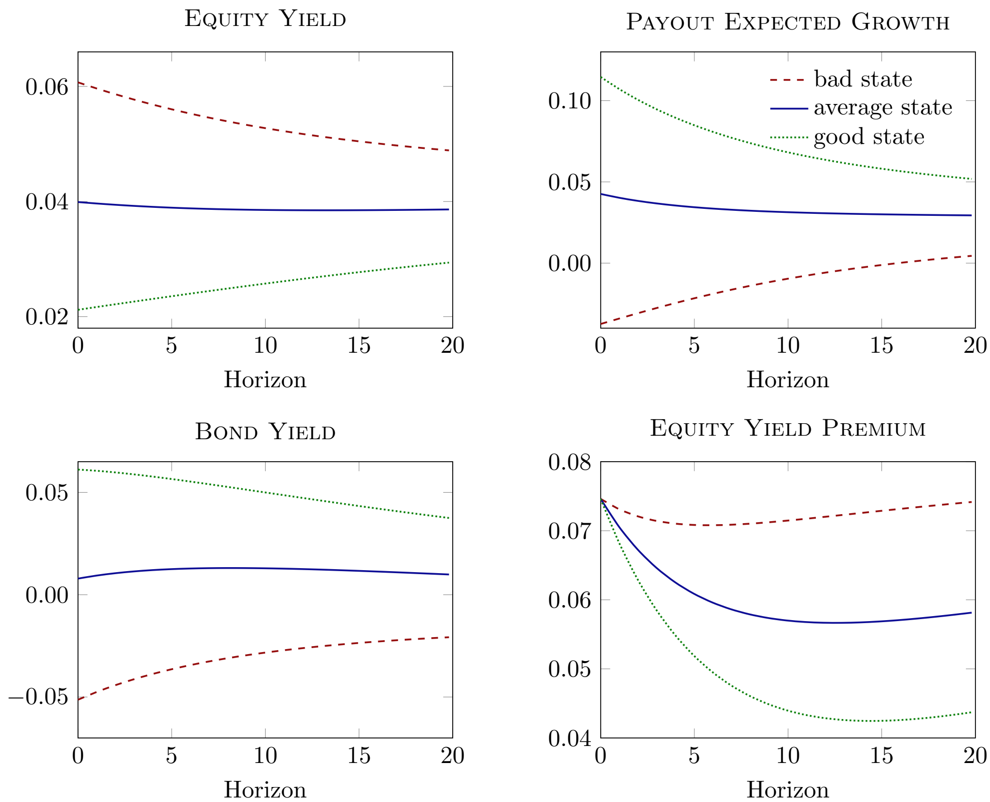

Figure 10: Equity Yield Decomposition. The panels report the model equity yield (upper, left), the model payout expected growth (upper, right), the model bond yield (lower, left), and the model equity yield premium (lower, right) as a function of the horizon for several values of EGV (x_t).

To better inspect the model mechanism, we exploit that the equity yield is the difference between the risk-free bond yield, $by(t,\tau)$, and expected payout growth, $g_D(t,\tau)$, plus the equity yield premium, $\vartheta(t,\tau)$:

$$ey(t,\tau) = by(t,\tau) - g_D(t,\tau) + \vartheta(t,\tau).$$

Figure 10 reports the equity yield and its three components in expansions, recessions, and the steady state. First, the premium component features countercyclical level and slope, because it represents a time-varying compensation for the exposure to EGV—affecting long-term payouts more heavily than short-term ones. Second, the bond is a hedge instrument against equity risk and, thus, the risk-free bond yield is procyclical in level. In turn, the slope of bond yields inherits the countercyclical dynamics of the risk premium slope. Third, expected payout growth is procyclical in level and has countercyclical slope because of the mean-reverting dynamics of both EGV and short-run shocks. As a result of these three forces, the equity yield is countercyclical in level—because the joint effect of expected growth and risk premium dominates the effect of the bond yield—but features procyclical slope—because the effect of expected growth dominates the joint effect of the bond yield and risk premium.[22]

IV.C Cross-Sectional Returns and Value Premium

We now study the model predictions for the cross-section of equity returns. We set $\varphi_V = 1, \varphi_G = -0.05$, and idiosyncratic risk $\sigma_\varphi = 8\%$, see Eq. (23) and Table 4.

The upper left panel of Figure 11 shows valuation ratios for both value and growth firms. The price of value firms is lower in level and more sensitive to EGV. The upper right panel of the figure displays the premium of the high-minus-low portfolio—that is, the value premium. The value premium is positive, sizable (about 3.5%), and countercyclical. These three results are consistent with the empirical literature and with our results in Section II: EGV is a driver of the value premium dynamics (claim *4* of our Proposition).

To better inspect the model mechanism, we consider the premium on the claim of value and growth payouts across the horizons. The long-run impact of EGV on payouts suggests that long-term claims to value-type payouts should feature a larger and more volatile premium than short-term claims. Instead, the lower loading of growth-type payouts to EGV suggests that both long-term claims and short-term claims should command a similar risk

[22]The forward equity yield can be written either as the difference between the equity yield and the risk-free bond yield or as the difference between the equity yield premium and expected payout growth: $fey(t,\tau) = ey(t,\tau) - by(t,\tau) = \vartheta(t,\tau) - g_D(t,\tau)$. In turn, the forward equity yield shows slightly sharper cyclicality of both level and slope than the equity yield, because it does not account for the opposite bond yield cyclicality of both level and slope. Consistently, Section II documents that forward equity yields and equity yields are very similar and their relation with EGV is indistinguishable.

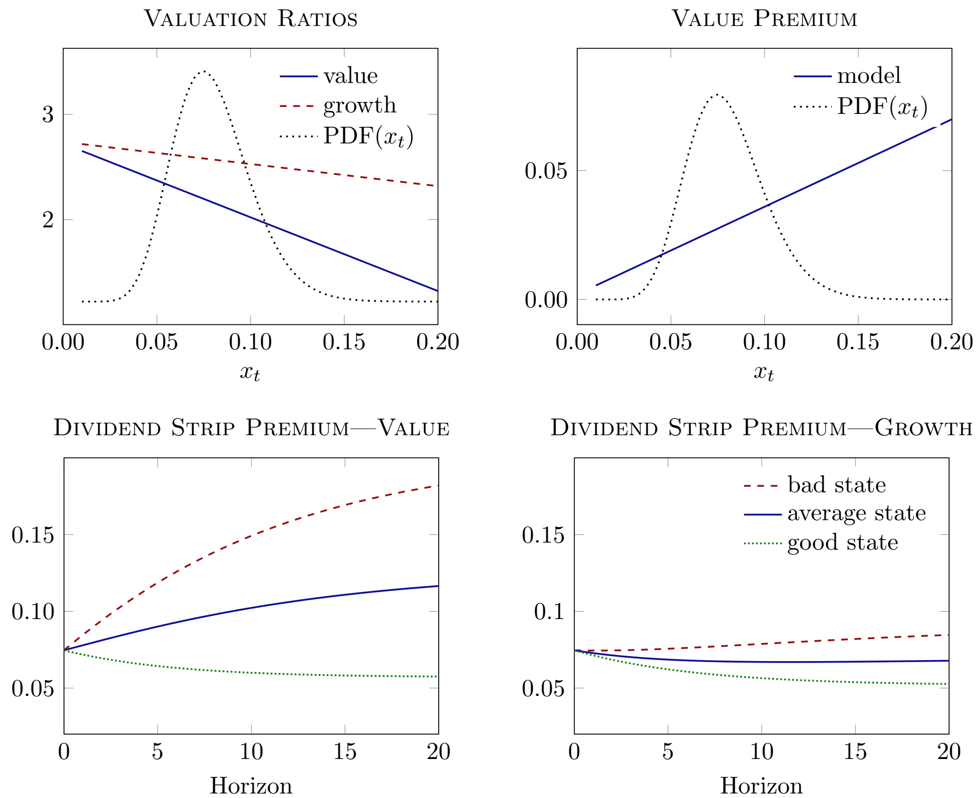

Figure 11: Value and Growth Price Dynamics and Term Structures. The upper panels of the figure report the model log price-payout ratio of value and growth firms (left), the model value premium (right) as a function of EGV (x_t). The lower panels of the figure show the model dividend strip risk premium for value (left) and growth firms (right) as a function of the horizon for several values of EGV (x_t).

premium and feature little sensitivity to EGV. The lower panels of Figure 11 display such heterogeneity in the dynamics of the premia to claims of value-type and growth-type payouts across the horizons. This result is consistent with the long-run predictability of value returns by EGV and the lack of predictability of growth returns, as documented in Section II.

The model mechanism of the value premium dynamics is interesting for three reasons. First, GKK21 document that the risk premia to claims of value- and growth-type payouts respectively increase and are flat or slighlty decrease with the horizon. Recently, Hasler et al. (2020) show that rational learning helps understand these unconditional slopes but they do not investigate dynamics. Our framework provides an alternative explanation for this stylized fact and reconciles it with the countercyclical dynamics of both the value premium and equity term premium. Second, we share with Bansal, Dittmar, and Lundblad (2005) the idea that the value premium arises from the excess loading of value firms on long-term fundamental risk over growth firms. However, their approach leads to negligible short-term risk premia, which are in contrast with the empirical evidence. Instead, our general equilibrium explains sizable short-term risk premia, similar to the partial equilibrium model of Lettau and Wachter (2007). Nevertheless, differently from their framework but consistent with recent empirical findings, our model does not predict downward-sloping (resp., upward-sloping) compensations to value (growth) firms. Third, the close connection between the model predictions about cross-sections and term structures with their empirical counterparts strongly corroborates the main model mechanism for the equity term premium dynamics.

V Robustness of the Model Mechanism

V.A Unconditional Equity Term Premium

BKK12 and Gormsen (2021) provide evidence that unconditional equity premia are downward-sloping. However, Bansal et al. (2021) call into question this result on the grounds that small samples can over-represent economic conditions in which the slope is negative (e.g., recessions) and, thus, lead to a wrong assessment about the unconditional slope.

While we are agnostic about the resolution of this empirical issue, we verify whether our economic mechanism is robust to the sign of the unconditional slope. In particular, we verify whether: (i) the model dynamics of equity slope are affected by the sign of the unconditional slope, and (ii) the model can reconcile standard asset pricing moments with the dynamics of the equity term structure under either positive or negative unconditional term premium.

In our model, the equity slope dynamics are robust to the sign of the unconditional slope. The dividend strip risk premium in Eq. (20) has two components: a permanent and heteroscedastic component driven by EGV (and commanding upward-sloping compensations) and a transitory and homoscedastic component due to short-run shocks (and commanding

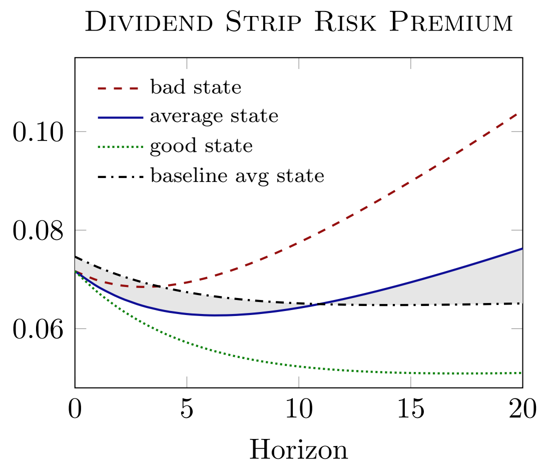

Figure 12: Unconditional Equity Term Premium. The figure reports the dividend strip risk premium as a function of the horizon for several values of EGV (x_t) under our alternative calibration. The shaded area denote the difference with respect to the baseline calibration in the steady-state.

downward-sloping compensations). The unconditional slope depends on the relative strength of the two components. However, the conditional slope moves with EGV and, so, features countercyclical dynamics and can switch sign over time.

We therefore consider an alternative calibration where we increase the persistence of EGV, x_t and decrease the persistence of the short-run shock, z_t. Namely, we change λ_x from 0.15 to 0.10 and λ_z from 0.15 to 0.20. We keep all the other parameters unchanged. Under this alternative calibration, standard asset pricing moments are still reasonable and similar to those under our baseline calibration. Appendix Table C9 reports several moment statistics from our model simulations. We observe a slightly higher equity premium and a lower risk-free rate as a result of preferences for the early resolution of uncertainty. Importantly, the change in the relative strength of the two shocks affects the risk premium across the horizon. Figure 12 shows the dividend strip risk premium at the average state under both the alternative and the baseline calibrations. Under this alternative calibration, the unconditional equity term premium shifts from negative to positive. Figure 12 also shows the dividend strip risk premium in good and bad states. As under the baseline calibration (see Figure 9) and in accord with the empirical evidence, compensations increase with the horizon in bad states and decrease with the horizon in good states. In turn, the equity term premium has countercyclical dynamics as documented by Gormsen (2021).

Overall, the economic mechanism of our model is robust to the sign of the unconditional equity term premium. Moreover, in accord with the empirical evidence, short-term assets command a high compensation even if the unconditional term premium is positive, as shown in Figure 12. Thus, our model addresses the challenge posed by the empirical findings in BBK12 to leading models, such as Campbell and Cochrane (1999), Bansal and Yaron (2004), and Wachter (2013)—which cannot explain sizable short-term risk premia, independently of the sign of the unconditional slope.

V.B Heteroscedastic Transitory Risk

A key ingredient of the baseline model is that expected growth volatility drives long-term premia. This is supported by our empirical analysis of Section II and implemented in the model through the assumption of heteroscedasticity and homoscedasticity of x_t and z_t, respectively. Although parsimonious, this assumption finds empirical support (see Appendix B) and is particularly convenient to illustrate the main model mechanism.

At the same time, we acknowledge that a limited fraction of transitory risk actually comoves with our EGV measure (see Appendix B). In this section, we show that including such a pattern in the model does not alter the predictions about the equity term structure and its dynamics. To this end, we simply replace the homoscedastic dynamics of z_t in Eq. (13) with a conditional variance that is affine in x_t:

$$dz_t = -\lambda_z z_t dt + \sqrt{\sigma_{z0}^2 + \sigma_{zx}^2\, x_t}\, dB_{z,t}. \tag{27}$$

where, in accord with the data, the term $\sigma_{zx}^2 \times \mathbb{E}[x_t]$ is a small fraction of the average conditional variance of z_t. Appendix D provides the model derivation, which is still obtained with the approach of Eraker and Shaliastovich (2008).

To illustrate the model predictions, we keep the parameters from the baseline calibration

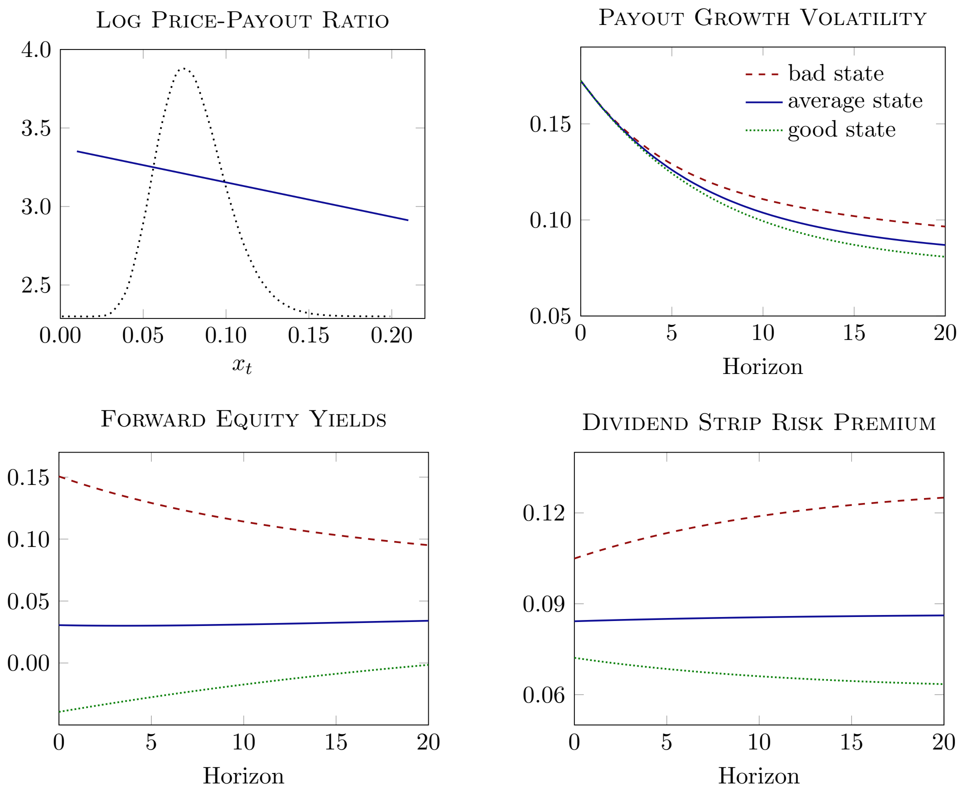

Figure 13: Main Model Predictions with Heteroscedastic Transitory Risk. The upper left panel reports the model log price to payout ratio as a function of EGV (x_t). The (standardized) unconditional density of EGV is superimposed. The upper right panel, the lower left panel, and the lower right panel report the volatility of payout growth rates, the forward equity yields and the dividend strip risk premium respectively, as a function of the horizon for several values of EGV (x_t).

of Table 3, with the exception of σ_z, which is replaced by the parameters $\sigma_{z0} = 0.023$ and $\sigma_{zx} = 0.036$. The model predicts a log price-payout ratio of 3.20, a risk-free rate of 0.9%, an equity premium of 7.4%, and a return volatility of 14.2%. These numbers compare quite well with both the data and the baseline model predictions. Figure 13 displays the log price-payout ratio as a function of x_t, the volatility of payout growth rates, the forward equity yields, and the dividend strip risk premium as a function of the horizon for several values of x_t. Consistent with the data and with the predictions of the baseline model, payout volatility markedly decreases with the horizon, the forward equity yields increase (decrease) with the horizon in good (bad) states, and the dividend strip risk premium decreases (increases) with the horizon in good (bad) states. Therefore, the model still leads to a procyclical equity yield slope and a countercyclical equity term premium. The state of the economy is measured by either expected growth volatility—in accord with our empirical findings—or the equilibrium price-payout ratio—endogenizing the findings of Gormsen (2021).

V.C Full Market Participation

While the baseline model assumes limited market participation, in this Section we discuss the implications of alternatively assuming full market participation. Under full market participation (FMP), the equilibrium can still be derived using a standard methodology (Eraker and Shaliastovich, 2008). Appendix D provides the model derivation. The main difference with respect to the baseline model is that the return on wealth, which enters the state-price density, should be computed for a representative agent that consumes total resources ($C = W + D$). Thus, the wealth of market participants $\mathcal{W}$ differs from equity and is instead a claim on the stream C:

$$\mathcal{W}_t^{\text{FMP}} = \mathbb{E}_t \int_0^\infty \frac{M_{t+\tau}^{\text{FMP}}}{M_t^{\text{FMP}}} C_{t+\tau} d\tau \neq P_t = \mathcal{W}_t^{\text{LMP}},$$

where the state-price density satisfies

$$d \log M_t^{\text{FMP}} = \theta \log \beta dt - \frac{\theta}{\psi} d \log C_t - (1-\theta) d \log R_{c,t},$$

and $R_{c,t}$ is the return on wealth. Since total resources C weigh on short-term risk less than payouts D, the equilibrium state-price density implies a lower price of risk for short-run shocks, relative to that for EGV, than under limited market participation (LMP):

$$\Omega_z^{FMP} = \frac{\gamma\psi + k_1(1-\gamma\psi}{1-k_1(1-\lambda_z)}\sigma_z < \Omega_z^{LMP}.$$

While the assumption about market participation affects the level of the term structure and the unconditional equity term premium, the model dynamics are not affected. When EGV rises, prices decline, equity yields drop, and the equity term premium increases.

Figure 14 displays the dynamics of the term structure of equity under full market participation. We observe that the model predictions about the equity term structure *dynam-*

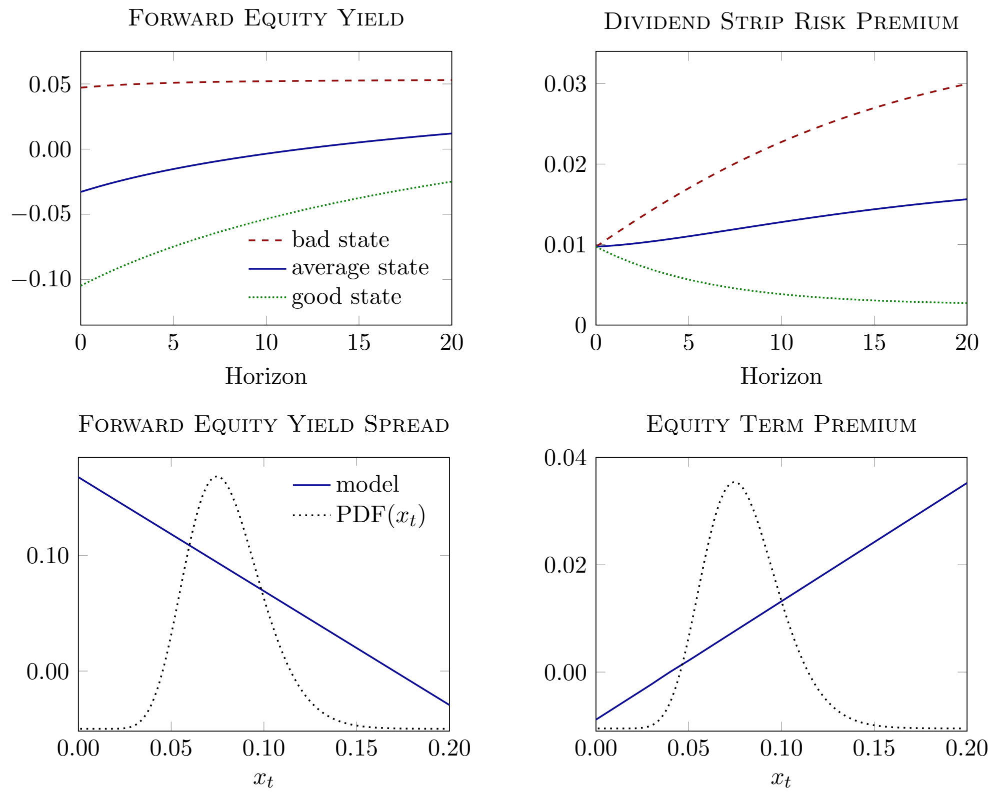

Figure 14: Equity Slope Dynamics under Full Market Participation. The top graphs of the figure report the model forward equity yield (left) and the model dividend strip risk premium (right) as a function of the horizon for several values of EGV (x_t). The bottom graphs of the figure show the model forward equity yield spread (left) and the model equity term premium (right) as a function of EGV (x_t). The (standardized) unconditional density of EGV (x_t) is superimposed.

ics—the core of our analysis—do not depend on the assumption about limited or full market participation. Indeed, the model still predicts a procyclical equity yield slope and a countercyclical equity term premium. However, full market participation affects the quantitiave model predictions. As discussed in Section IV, under limited market participation, the model reconciles (i) the timing of fundamental risk, (ii) standard moments (e.g., equity premium and risk-free rate puzzles), (iii) sizable short-term risk premia, and (iv) the dynamics of the term structure of equity. Instead, under full market participation and the same parameter setting, the model continues to describe well the timing of fundamental risk and the equity term structure cyclicality, but it does not predict sizable equity compensations at any horizon. Alternative parameter settings under full market participation predict either a high equity premium or downward-sloping payout risk and sizable short-term compensations. These results suggest that the empirical observation of limited market participation is an important ingredient to study the term structure of equity.[23]

VI Conclusion

A parsimonious general equilibrium framework provides a comprehensive understanding of the recent empirical findings regarding the term structure of equity, its dynamics, its implications for the cross section of returns, and in particular its link with macroeconomic risk. The economic mechanism relies on the interaction between two risks affecting economic fundamentals: permanent shocks driven by expected growth volatility—which induce upward-sloping risk with the horizon—and transitory shocks capturing stationary fluctuations—which induce downward-sloping risk. In equilibrium, the interaction and the relative magnitude of these two risks determine the slope dynamics of equity compensations. Extensive empirical evidence supports the model assumptions, economic mechanism, and predictions.

Our model jointly generates the observed features of the term structure of equity, including a potentially negative unconditional slope of term premia, countercyclical variation of term premia, and procyclical variation of equity yields. The model also explains premia to value (respectively, growth) stocks, which are increasing (flat or slighlty decreasing) with the horizon. At the same time, our model performs well in capturing standard asset pricing moments under realistic assumptions about the economic environment (e.g., consumption and payouts cointegration and limited market participation) and standard preferences.

[23]While endogenous participation goes beyond the scope of the paper, our analysis points to it as an unexplored economic channel to reconcile equity risk premia across the horizons.

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A Model Derivation and Proofs

Affine Notation

The vector $X_t = (y_t, x_t, z_t, w_t)^\intercal$ collects the two state variables of our model x_t and z_t as well as accumulated expected growth $y_t = \mu t + \int_0^t (\bar{x} - x_s) ds$ and the specific component of cross-sectional payouts $w_t = \varphi \int_0^t (\bar{x} - x_s) ds + \sigma_\varphi B_{\varphi,t}$. The vector belongs to the affine class and has dynamics:

$$\begin{aligned} dX_t &= \mu(X_t) dt + \Sigma(X_t) d\mathcal{B}_t, \\ \mu(X_t) &= \mathcal{M} + \mathcal{K} X_t, \\ \Sigma(X_t)\Sigma(X_t)^\intercal &= h + \sum\nolimits_{i \in \{y,x,z,w\}} H_i X_{i,t}, \end{aligned}$$

with Brownian motions $\mathcal{B}_t = (B_{y,t}, B_{x,t}, B_{z,t}, B_{w,t})^\intercal$ and the following coefficients:

$$\mathcal{M} = \begin{pmatrix} \mu + \bar{x} \\ \lambda_x \bar{x} \\ 0 \\ \varphi \bar{x} \end{pmatrix}, \qquad \mathcal{K} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -\lambda_x & 0 & 0 \\ 0 & 0 & -\lambda_z & 0 \\ 0 & -\varphi & 0 & 0 \end{pmatrix},$$

$$h = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_\varphi^2 \end{pmatrix}, \qquad H_y = H_z = H_w = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad H_x = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The following selection vectors allow to recover consumption, aggregate payouts, and cross-sectional payouts:

$$\begin{aligned} v_C &= (1, 0, 1, 0)^\intercal &\Rightarrow\quad & v_C^\intercal X_t = \log C_t, \\ v_D &= (1, 0, \phi, 0)^\intercal &\Rightarrow\quad & v_D^\intercal X_t = \log D_t, \\ v_\varphi &= (1, 0, \phi, 1)^\intercal &\Rightarrow\quad & v_\varphi^\intercal X_t = \log D_t^\varphi. \end{aligned}$$

Moment Generating Function

The following conditional expectation allows to compute the moment generating function for the logarithm of C, D, and D^φ at any future horizon τ:

$$\mathbb{E}_t[\exp(\mathbf{u}^\intercal X_{t+\tau})] = \exp(\bar{b}_0(\tau) + \bar{b}(\tau)^\intercal X_t). \tag{A1}$$

As shown in Duffie, Pan, and Singleton (2000), the functions $\bar{b}_0(\tau)$ and $\bar{b}(\tau) = \big(\bar{b}_y(\tau), \bar{b}_x(\tau), \bar{b}_z(\tau), \bar{b}_w(\tau)\big)^\intercal$ solve the following system of ODE's:

$$\begin{aligned} \bar{b}_0'(\tau) &= \mathcal{M}^\intercal \bar{b}(\tau) + \frac{1}{2} \bar{b}(\tau)^\intercal h \bar{b}(\tau), \\ \bar{b}'(\tau) &= \mathcal{K}^\intercal \bar{b}(\tau) + \frac{1}{2} \bar{b}(\tau)^\intercal H \bar{b}(\tau). \end{aligned}$$

By setting the initial conditions $\bar{b}_0(0) = 0$ and either $\bar{b}(0) = uv_C$, $\bar{b}(0) = uv_D$, or $\bar{b}(0) = uv_\varphi$, Eq. (A1) computes the time-t conditional expectation of either $C_{t+\tau}, D_{t+\tau}$, or $D^{\varphi}_{t+\tau}$ with power u respectively. These expectations can be used to build the term structure of growth rates volatility.

Equity and State-Price Density

We follow Eraker and Shaliastovich (2008) and the state-price density based on recursive preferences of Epstein and Zin (1989) type. To do so, we use the Campbell and Shiller (1988) approximation to log-linearize the return R_t on the wealth of market participants (that in our economy corresponds to the equity market and pays-out D_t):

$$d\log R_t = k_0 dt + k_1 dpd_t - (1-k_1)pd_t dt + d\log D_t, \tag{A2}$$

Where k_0 and k_1 are endogenous constants to be determined. We conjecture that the log price-payout ratio (which is also the log wealth-consumption ratio of market participants) is an affine function of X_t: $pd_t = A_0 + A^\intercal X_t$.

We use Eq. (A2) to rewrite the state-price density dynamics as follows:

$$\begin{aligned} d\log M_t =& \theta \log\beta dt - \frac{\theta}{\psi} d\log D_t - (1-\theta) d\log R_t \\ =& \left(\theta\log\beta - (\theta-1)\log k_1 + (\theta-1)(k_1-1)A^\intercal (X_t - \mu_X)\right) dt - \lambda^\intercal dX_t, \end{aligned} \tag{A3}$$

where $\lambda = \gamma v_D + (1-\theta) k_1 A$ and $\mu_X = (0, \bar{x}, 0, 0)^\intercal$. Then, the Euler equation can be written as:

$$1 = \mathbb{E}_t \left[\frac{M_{t+\tau}}{M_t} e^{\int_0^\tau d\log R_{t+s}} \right], \qquad \forall\tau.$$

Since the term in the conditional expectation has to be a martingale, we apply Itô's lemma to compute its drift that we set equal to zero:

$$0 = \theta\log\beta + \chi^\intercal (\mathcal{M} + \mathcal{K}X_t) + \theta k_0 - \theta(1-k_1)(A_0 + A^\intercal X_t) + \frac{1}{2}\chi\Sigma(X_t)^\intercal\Sigma(X_t)\chi^\intercal X_t, \tag{A4}$$

where $\chi = \theta\left(\left(1 - \frac{1}{\psi}\right) v_D + k_1 A\right)$. Since Eq. (A4) holds for all X_t and we set the coefficients on X_t and the residual constant equal to zero. The endogenous coefficients k_1, A_0 and $A = (A_y, A_x, A_z, A_w)^\intercal$ are obtained by solving the following system:

$$\begin{aligned} 0 =& \mathcal{K}^\intercal\chi - \theta(1-k_1)A + \frac{1}{2}\chi^\intercal H\chi, \\ 0 =& \theta\left(\log\beta + k_0 - (1-k_1)A_0\right) + \mathcal{M}^\intercal\chi + \frac{1}{2}\chi^\intercal h\chi, \\ \theta\log k_1 =& \theta\left(\log\beta + (1-k_1)A^\intercal\mu_X\right) + \mathcal{M}^\intercal\chi + \frac{1}{2}\chi^\intercal h\chi. \end{aligned}$$

The solution coefficients should be inserted into (A3) to obtain the equilibrium state price density.[24] The equity price is then given by $P_t = D_t \exp(A_0 + A^\intercal X_t)$, where $A_y = A_w = 0$.

Applying Itô's Lemma to (A3) yields:

$$\begin{aligned} \frac{dM_t}{M_t} &= \left(\theta \log \beta - (\theta - 1) \log k_1 + (\theta - 1)(k_1 - 1) A^\intercal (X_t - \mu_X) + \mu(X_t)^\intercal \lambda\right) dt \\ &\quad + \frac{1}{2} \lambda^\intercal \Sigma(X_t) \lambda dt - \lambda^\intercal \Sigma(X_t) d\mathcal{B}_t \\ &= -\left(r_0 + \bar{r}^\intercal X_t\right) dt - \lambda^\intercal \Sigma(X_t) d\mathcal{B}_t, \end{aligned} \tag{A5}$$

where the coefficients r_0 and $\bar{r} = (r_y, r_x, r_z, r_w)^\intercal$ are:

$$\begin{aligned} r_0 &= -\theta \log \beta + (\theta - 1)\left(\log k_1 + (k_1 - 1) A^\intercal \mu_X\right) + \mathcal{M}^\intercal \lambda - \frac{1}{2} \lambda^\intercal h \lambda, \\ \bar{r} &= (1 - \theta)(k_1 - 1) A + \mathcal{K}^\intercal \lambda - \frac{1}{2} \lambda^\intercal H \lambda. \end{aligned}$$

Therefore, the risk-free rate is given by $r_t = r_0 + \bar{r}^\intercal X_t$ and the vector of risk prices is given by $\Omega(X_t) = (\Omega_y, \Omega_x, \Omega_z, \Omega_w)^\intercal = \Sigma(X_t)^\intercal \lambda$, where it turns out that $r_y = r_w = 0$ and $\Omega_y = \Omega_w = 0$. Consequently, the risk premium on equity is equal to $RP(x_t) = ((A + v_D)^\intercal \Sigma(X_t))\Omega(X_t)$, which is an affine function of x_t only.

Term Structures

Following Duffie et al. (2000), the risk-neutral dynamics of X_t are given by:

$$\begin{aligned} dX_t &= \left(\mathcal{M}^{\mathcal{Q}} + \mathcal{K}^{\mathcal{Q}} X_t\right) dt + \Sigma(X_t) d\mathcal{B}_t^{\mathcal{Q}}, \\ \mathcal{M}^{\mathcal{Q}} &= \mathcal{M} - h\lambda, \\ \mathcal{K}^{\mathcal{Q}} &= \mathcal{K} - H\lambda, \\ d\mathcal{B}_t^{\mathcal{Q}} &= d\mathcal{B}_t + \Sigma(X_t)^\intercal \lambda \, dt. \end{aligned}$$

Then, we can compute the discounted value of several payouts, such as $D_{t+\tau}, D_{t+\tau}^{\varphi}$, and the unitary payout of a risk-less bond:

$$\mathbb{E}_t \left[\frac{M_{t+\tau}}{M_t} \exp(v^\intercal X_{t+\tau})\right] = \mathbb{E}^{\mathcal{Q}} \left[\exp(-\int_0^\tau r_{t+s} ds + v^\intercal X_{t+\tau})\right] = \exp(q_0(\tau) + q(\tau)^\intercal X_t),$$

where $v \in \{v_D, v_\varphi, (0,0,0,0)^\intercal\}$. The deterministic function $q_o(\tau)$ and $q(\tau) = (q_y(\tau), q_x(\tau), q_z(\tau), q_w(\tau))$ solve the following system of ODE's:

$$\begin{aligned} q_0'(\tau) &= -r_0 + \left(\mathcal{M}^{\mathcal{Q}}\right)^\intercal q(\tau) + \frac{1}{2} q(\tau)^\intercal h \, q(\tau), \\ q'(\tau) &= -\bar{r} + \left(\mathcal{K}^{\mathcal{Q}}\right)^\intercal q(\tau) + \frac{1}{2} q(\tau)^\intercal H q(\tau), \end{aligned}$$

[24] Note that the above system of equations could yields multiple solutions. Tauchen (2011) proposes to select the root which ensures the non-explosiveness of the system. Alternatively, one could select an economically reasonable solution.

with initial conditions $q_0(0) = 0$ and $q(0) = v$.

Therefore, the strip price of the aggregate payout, the risk-less bond price, and the strip price of the cross-sectional payout are given by

$$
\begin{aligned}
P_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = v_D, \\
B_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = (0,0,0,0)^\intercal, \\
P^\varphi_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = v_\varphi.
\end{aligned}
$$

For the strip price of the aggregate payout in Eq. (19), the price elasticities to the permanent and short-run shocks satisfy:

$$
\begin{aligned}
a_x(\tau) &= \frac{2\Psi_0}{\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2}\coth\left(\frac{\tau}{2}\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2}\right) - \Psi_1}, \\
a_z(\tau) &= \frac{\phi}{\psi} + \phi(1 - 1/\psi)e^{-\lambda_z\tau},
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi_0 &= \gamma - 1 + A_x(1 - k_1(1-\lambda_x))\frac{\gamma - 1/\psi}{1 - 1/\psi} + A_x^2\frac{k_1^2\sigma_x^2(\gamma - 1/\psi)^2}{2(1-1/\psi)^2}, \\
\Psi_1 &= -k_1\sigma_x^2 A_x\frac{\gamma - 1/\psi}{1 - 1/\psi} - \lambda_x, \\
\Psi_2 &= \frac{\sigma_x^2}{2}.
\end{aligned}
$$

For the risk-less bond price in Eq. (22), the price elasticities to the permanent and short-run shocks satisfy:

$$
\begin{aligned}
b_x(\tau) &= \frac{2\Phi_0}{\sqrt{\Phi_1^2 - 4\Phi_0\Phi_2}\coth\left(\frac{\tau}{2}\sqrt{\Phi_1^2 - 4\Phi_0\Phi_2}\right) - \Phi_1}, \\
b_z(\tau) &= \frac{\phi}{\psi}\left(1 - e^{-\lambda_z\tau}\right).
\end{aligned}
$$

where $\Phi_0 = \Psi_0 + 1$, $\Phi_1 = \Psi_1$, and $\Phi_2 = \Psi_2$. For the strip price of the cross-sectional payout in Eq. (26), the price elasticities to the permanent and short-run shocks satisfy:

$$
\begin{aligned}
a^\varphi_x(\tau) &= \frac{2\Theta_0}{\sqrt{\Theta_1^2 - 4\Theta_0\Theta_2}\coth\left(\frac{\tau}{2}\sqrt{\Theta_1^2 - 4\Theta_0\Theta_2}\right) - \Theta_1}, \\
a^\varphi_z(\tau) &= \frac{\phi}{\psi} + \phi(1 - 1/\psi)e^{-\lambda_z\tau},
\end{aligned}
$$

where $\Theta_0 = \Psi_0 - \varphi$, $\Theta_1 = \Psi_1$, and $\Theta_2 = \Psi_2$.

Cross-Sectional Equity

The price of the stock paying out the stream D_t^φ can be computed either as the time integral of the corresponding strip price over any maturity or via an exponential affine approximation. Such exponential affine approximation is given by

$$P_t^\varphi = D_t^\varphi \exp(A_0^\varphi + (A^\varphi)^\intercal X_t).$$

The coefficients A_0^φ, $A^\varphi = (A_y^\varphi, A_x^\varphi, A_z^\varphi, A_w^\varphi)^\intercal$, and the endogenous constant k_1^φ solve the following system:

$$\begin{aligned}
0 =& (\theta - 1)(k_1 - 1)A + (k_1^\varphi - 1)A^\varphi + \mathcal{K}^\intercal \chi_\varphi + (1/2)\chi_\varphi^\intercal H \chi_\varphi, \\
0 =& \theta \log \beta - (\theta - 1)(\log k_1 + (k_1 - 1)A^\intercal \mu_X) - (\log k_1^\varphi + (k_1^\varphi - 1)(A^\varphi)^\intercal \mu_X) + \mathcal{M}^\intercal \chi_\varphi \\
&+ (1/2)\chi_\varphi^\intercal h \chi_\varphi, \\
0 =& A_0^\varphi + (A^\varphi)^\intercal \mu_X - \log k_1^\varphi + \log(1 - k_1^\varphi),
\end{aligned}$$

where $\chi_\varphi = v_\varphi + k_1^\varphi A^\varphi - \lambda$. It turns out that $A_y^\varphi = A_w^\varphi = 0$. Therefore, the risk premium on the cross-sectional stock is equal to $RP^\varphi(x_t) = ((A^\varphi + v_\varphi)^\intercal \Sigma(X_t))\Omega(X_t)$, which is an affine function of x_t only.

State-Price Density Decomposition

We follow Alvarez and Jermann (2005) and Hansen and Scheinkman (2009) and decompose the equilibrium state-price density in its permanent (martingale) component and its transitory component. The logarithm of the state-price density equals:

$$\log M_t = -\int_0^t (r_s + \frac{1}{2}(\Omega_x^2(x_s) + \Omega_z^2))ds - \int_0^t \Omega_x(x_s)dB_{x,s} - \Omega_z B_{z,t},$$

with $\log M_0 = 0$.

Example 6.2 in Hansen and Scheinkman (2009) nests the above functional form. The permanent (martingale) component $\widehat{M_t}$ of the state-price density is given by

$$\begin{aligned}
\log \widehat{M_t} =& -\frac{1}{2}(c_x\sigma_x - \Omega_x)^2 \int_0^t x_s ds - \frac{1}{2}(c_z\sigma_z - \Omega_z)^2 t \\
&- (c_x\sigma_x - \Omega_x)\int_0^t \sqrt{x_s} dB_{x,s} - (c_z\sigma_z - \Omega_z)B_{z,t},
\end{aligned}$$

where

$$\begin{aligned}
c_x &= \frac{\lambda_x + \sigma_x\Omega_x - \sqrt{2r_x\sigma_x^2 + (\lambda_x + \sigma_x\Omega_x)^2}}{\sigma_x^2}, \\
c_z &= -\frac{r_z}{\lambda_z},
\end{aligned}$$

with $\Omega_x = \Omega_x(x_t)/\sqrt{x_t}$. This decomposition allows us to verify that the state-price density satisfies the bound introduced by Alvarez and Jermann (2005), as reported in Section IV.A.

Proposition Proof

Our results are valid under the parametric restriction $\sigma_x^2 < \overline{\sigma}_x^2$, where

$$\overline{\sigma}_x^2 = \frac{\psi\lambda_x \left(2\gamma\psi + k_1\left(-2\gamma\psi + (\gamma\psi+\psi-2)\lambda_x + 2\right) - 2\right)}{2k_1(\gamma\psi-1)^2}. \tag{A6}$$

In the steady-state, the equity premium equals 9.5% at the upper boundary $\sigma_x^2 = \overline{\sigma}_x^2$ in our baseline calibration. Since the equity premium is much lower in the data, the above constraint is never binding in our analysis. Other parameter restrictions we impose are $0 < k_1 < 1$, $0 < \lambda_x < 1$, $\gamma > \psi > 1$.

Lemma 1. $\Psi_0 < 0$.

Proof. Note that Ψ_0 is strictly increasing in σ_x^2:

$$\frac{\partial\Psi_0}{\partial\sigma_x^2} = \frac{2k_1^2(\psi-1)(\gamma\psi-1)\left(k_1\lambda_x - k_1 + 1\right)}{\Xi\psi^2\left(k_1\left(\lambda_x-1\right)+\Xi+1\right)^2} > 0. \tag{A7}$$

Furthermore, setting $\sigma_x^2 = \overline{\sigma}_x^2$ (i.e., to its highest value given our constraint) yields $\Psi_0 < 0$. Since Ψ_0 is strictly increasing in σ_x^2, this implies that for $\sigma_x^2 < \overline{\sigma}_x^2$, Ψ_0 is negative. □

Lemma 2. $\Psi_1 < 0$.

Proof. Note that Ψ_1 is strictly increasing in σ_x^2:

$$\frac{\partial\Psi_1}{\partial\sigma_x^2} = \frac{k_1(\gamma\psi-1)}{\Xi\psi} > 0. \tag{A8}$$

Furthermore, setting $\sigma_x^2 = \overline{\sigma}_x^2$ (i.e., to its highest value given our constraint) yields $\Psi_1 = 0$. Since Ψ_1 is strictly increasing in σ_x^2, this implies that for $\sigma_x^2 < \overline{\sigma}_x^2$, Ψ_1 is negative. □

Lemma 3. $A_x < 0$.

Proof. Since $A_x = -\frac{2(1-1/\psi)}{1-k_1(1-\lambda_x)+\Xi}$ and $\psi > 1$, we simply need to verify that:

$$1 - k_1(1-\lambda_x) + \Xi > 0. \tag{A9}$$

Since Ξ is a square root term and non-negative and since $0 < k_1 < 1$ and $0 < \lambda_x < 1$, this condition is satisfied. □

Part 1: Valuation ratios decrease with EGV:

We compute the derivative term:

$$\frac{\partial}{\partial x_t} \log \frac{P_t}{D_t} = \frac{A_x}{D_t} \exp\left(A_0 + A_x x_t + A_z z_t\right).$$

The above term is negative since A_x is negative as shown in Lemma 3. $\square$

Part 2: The slope of equity yields decreases with EGV:

We compute the cross-derivative term:

$$\frac{\partial^2}{\partial \tau \partial x} ey(t,\tau) = \frac{a_x(\tau)}{\tau^2} - \frac{\frac{\partial a_x(\tau)}{\partial \tau}}{\tau} = -\frac{2\Psi_0\left(\omega \operatorname{cosech}\omega^2 - \coth\omega\right)\left(\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2} + \Psi_1\right)}{\tau^2\left(\Psi_1 - \coth\omega\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2}\right)^2}. \tag{A10}$$

To determine the sign of (A10), note that the denominator contains a squared (real) term, ans is therefore positive. We therefore focus on determining the sign of the numerator. The term $\omega = \frac{\tau}{2}\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2}$ is strictly positive since $\Psi_0 < 0$, $\Psi_1 < 0$, $\Psi_2 > 0$, and $\tau > 0$, Consequently, the trigonometric expressions satisfy $\omega \operatorname{cosech}\omega^2 - \coth\omega < 0$ and $\coth\omega > 1$. Additionally considering $\Psi_0 < 0$ and the minus sign in front of the expression, it suffices to show that the following term is *positive*:

$$\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2} + \Psi_1.$$

Since $\Psi_0 < 0$ and $\Psi_2 > 0$, this condition is satisfied. $\square$

Part 3: The slope of the dividend strip risk premium increases with EGV:

We compute the cross-derivative term:

$$\frac{\partial^2}{\partial \tau \partial x} RP_{DS}(x_t,\tau) = \frac{A_x k_1 \sigma_x^2\left(\gamma - \frac{1}{\psi}\right)}{1 - \frac{1}{\psi}} \frac{\partial}{\partial \tau} a_x(\tau). \tag{A11}$$

Given $0 < k_1 < 1$, $\gamma > \psi > 1$ and $A_x < 0$, a sufficient condition for showing that (A11) is negative is that $\partial a_x(\tau)/\partial\tau$ is negative, where

$$\frac{\partial}{\partial \tau} a_x(\tau) = \frac{\Psi_0\left(\Psi_1^2 - 4\Psi_0\Psi_2\right)\left(\operatorname{cosech}\omega\right)^2}{\left(\Psi_1 - \coth\omega\sqrt{\Psi_1^2 - 4\Psi_0\Psi_2}\right)^2}.$$

Considering that the expression contains two squared and hence positive terms, it suffices to show that the following term is *negative*:

$$\Psi_0\left(\Psi_1^2 - 4\Psi_0\Psi_2\right).$$

Since, $\Psi_0 < 0$ and $\Psi_2 > 0$, this condition is satisfied. $\square$

Part 4: The value premium increases with EGV:

The coefficients A_x^φ and A_z^φ are derived using Eraker and Shaliastovich (2008):

$$A_x^\varphi = \Gamma_0 + \Gamma_1 \sqrt{\Gamma_2 - \frac{2(\psi-1)^2\varphi}{\Gamma_1}} \quad \text{and} \quad A_z^\varphi = -\frac{\phi\lambda_z(1-1/\psi)}{1-k_1^\varphi(1-\lambda_z)},$$

where Γ_0, Γ_1, and Γ_2 are quantities that are a function of σ_x, k_1, ψ, k_1^φ, γ, Ξ, and λ_x, are independent of φ or x_t. Specifically:

$$\Gamma_0 = \frac{(\gamma-1)k_1(\psi-1)\psi^2 - k_1^\varphi(\psi-1)\psi(\gamma\psi + k_1(\lambda_x-1)(\psi-1)-1) + k_1^\varphi\Xi(\gamma\psi-1)}{(\gamma-1)k_1(k_1^\varphi)^2\sigma_x^2(\psi-1)\psi^2},$$

$$\Gamma_1 = -\frac{1}{(k_1^\varphi)^2\sigma_x^2(\psi-1)^2},$$

$$\Gamma_2 = \frac{(\psi-1)^3\left(k_1^2(\psi-1)\psi\left((\gamma-1)\psi-(\gamma-1)(k_1^\varphi)^2\psi\left(2(\gamma-1)\sigma_x^2-(\lambda_x-1)^2\right)+2k_1^\varphi(\lambda_x(1-\psi)+\psi-1)\right)\right)}{(\gamma-1)k_1^2\psi^2}$$
$$+ \frac{2k_1k_1^\varphi(\gamma\psi-1)((\psi-1)\psi(k_1^\varphi(\lambda_x-1)-1)+\Xi) - 2(k_1^\varphi)^2(\gamma\psi-1)\left(\Xi-\psi^2+\psi\right)}{(\gamma-1)k_1^2\psi^2}.$$

It is now sufficient to show the following cross-derivative of (25) is positive:

$$\frac{\partial^2}{\partial\varphi\partial x}RP^\varphi(x_t) = \sigma_x^2\left(\frac{k_1A_x(\gamma-1/\psi)}{1-1/\psi}\right)\frac{\partial A_x^\varphi}{\partial\varphi} = \sigma_x^2\left(\frac{k_1A_x(\gamma-1/\psi)}{1-1/\psi}\right)\frac{-(\psi-1)^2}{\sqrt{\Gamma_2 - \frac{2\varphi(\psi-1)^2}{\Gamma_1}}}.$$

Since for $\sigma_x^2 > 0, \gamma > \psi > 1, k_1 > 0$, this condition is satisfied if:

$$\frac{A_x}{\sqrt{\Gamma_2 - \frac{2\varphi(\psi-1)^2}{\Gamma_1}}} < 0.$$

Since $A_x < 0$ from Lemma (3), and since we assume our model is well-defined (no imaginary quantities), the denominator is positive and hence the above condition is satisfied. $\square$

B Empirical Properties of the Transitory Component of Economic Growth

Construction of the Transitory Process

The representation of Eq. (3)-(9) for the long- and short-run components of economic growth naturally links to the model dynamics in Eq. (11)-(13). In this section, we compute and analyze the properties of the process "transitory$_{t+1}$", which corresponds to z_t in the model.

First, we recover the empirical counterpart to the shock $\sigma_z dB_{z,t}$ in Eq. (13) by taking the forecast error, defined as the difference between actual GDP growth and the SPF mean forecast, i.e., $\epsilon_{g,t+1} = g_{t+1} - f_{t,t+1}$ (see Eq. (3)). Second, we obtain a proxy for the drift term of aggregate resources (corresponding to expected growth), namely $\mathbb{E}_t[d \log C_t] = (\mu + \bar{x} - x_t - \lambda_z z_t)dt$. To this end, we filter the permanent component out of expected growth, as measured by the SPF mean forecast, by implementing the regression in Eq. (5) where $f_{t,t+1}$ is regressed on $\sigma_{t,t+1}$. The residual $e_{f,t}$ from such a specification captures the expected transitory component of growth $\mathbb{E}_t[dz_t] = -\lambda_z z_t dt$. Following Eq. (8), we can then compute the transitory shock to growth at time $t+1$ by summing $\epsilon_{g,t+1}$ and $e_{f,t}$, thus proxying for dz_t. By the same token, we build our model-based proxy for the z_t process as:

$$\text{transitory}_{t+1} = \sum_{s \le t+1} \Delta\text{transitory}_s = \sum_{s \le t} (\epsilon_{g,s+1} + e_{f,s}). \tag{B1}$$

The above construction then derives the transitory component under the assumption that our EGV measure drives the permanent component of expected growth (see Eq. (5)). An immediate validity check boils down to comparing the transitory component against the output gap —a well known proxy of the transitory component of economic growth. Appendix Figure B1 shows that transitory shocks and changes in output gap strongly comove over the entire sample 1968-2019 (with a correlation of 95.7%).

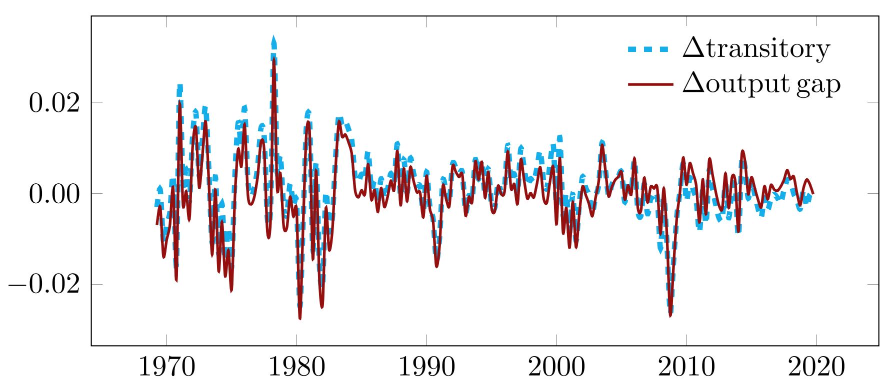

Figure B1: Transitory Shocks and Output Gap. This figure plots the changes of the transitory process from Eq. (B1) and the changes of output gap.

The Brownian shocks in Eq. (12)-(13) are independent. This assumption finds support in the data. We build the empirical counterparts of the Brownian shocks $dB_{x,t}$ and $dB_{z,t}$ as the innovations to the processes $\sigma_{t,t+1}$ and "transitory$_{t+1}$", respectively (for which we assume, in accord with the model, a square-root and a linear autoregressive dynamics discretized at quarterly frequency). We find that the correlation between these innovations is not statistically different from zero. The same result obtains when replacing "transitory$_{t+1}$" by the output gap. Below, we further analyze the properties of the transitory component.

Stationarity, Heteroscedasticity, and Cyclicality

In Appendix Table B1, we test for the stationarity of transitory$_{t+1}$ by means of Augmented Dickey-Fuller (ADF) tests. The objective is to verify if the unit root of economic growth is indeed driven by the permanent component in Eq. (7) alone. Due to uncertainty as to the specification of ADF tests, we define the underlying model in a less restrictive way by including a drift term (Dolado, Jenkinson, and Sosvilla-Rivero, 1990). In column 1, we allow for the presence of a lag in the specification, whereas in column 2 we select the optimal number of lags by means of the Akaike information criterion. In each case, we are able to reject the null hypothesis of the presence of a unit root in the transitory component of economic growth. This is reassuring for our modeling assumptions, especially in the light of the well-known low statistical power issues of ADF tests.

Table B1: Stationarity of the Transitory Process

	(1)	(2)
ADF lags	1	12
ADF stat	-2.29	-2.64
p-value	0.01	0.00
Observations	205	205

Note. This table reports results from tests of stationarity of the transitory process in Eq. (B1). ADF tests with a drift in the underlying model are presented. The number of lags included in the ADF specification is equal to one in column 1, whereas it is optimally selected according to the Akaike information criterion in column 2.

The process in Eq. (13) assumes a constant volatility σ_z for z_t, whereas the volatility of x_t in Eq. (12) is time-varying and defined as $\sigma_x\sqrt{x_t}$. In other words, we assume z_t and x_t to be homoscedastic and heteroscedastic, respectively. To test such assumptions, in Appendix Figure B2 we plot the residuals against the fitted values from AR(1) models for the transitory process—which proxies for z_t—and EGV (σ)—which proxies for x_t. In line with the assumption of homoscedasticity of z_t, the left graph displays no clear pattern, with the residuals pretty evenly scattered over the plot. By contrast, the right graph for EGV clearly shows that the dispersion of residuals increases with the fitted values, pointing to the heteroscedasticity of x_t.

To further investigate our assumption of homoscedasticity of z_t, we compute the conditional volatility of "transitory$_{t+1}$" as the four-quarter moving average of $|\epsilon_{g,t+1}|$ and verify its dependence on the permanent growth process as proxied by EGV by means of a linear regression (untabulated). We find that EGV loads positively and significantly, pointing to some degree of heteroscedasticity in "transitory$_{t+1}$", which is nonetheless of modest economic magnitude. Whereas this result corroborates the homoscedastacity of z_t, in Section V.B we present a model extension allowing for this form of heteroscedasticity. The main model mechanism is not affected and the model predictions remain essentially unchanged.

Finally, in Appendix Table B2 we assess the cyclicality of "transitory$_{t+1}$" by estimating the same specifications seen for EGV in Table 1. As we would expect, "transitory$_{t+1}$" appears to be strongly procyclical irrespective of the business or financial cycle measure we consider.

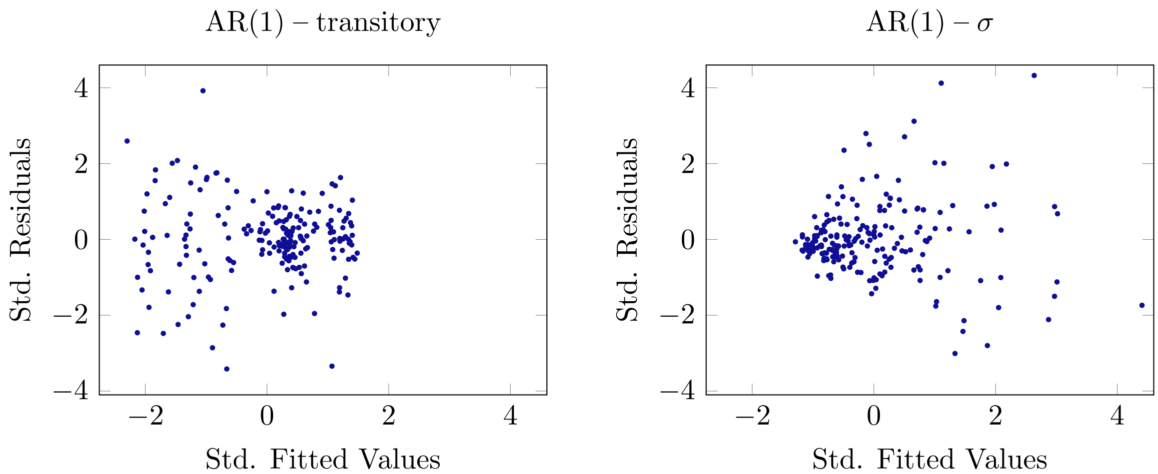

Figure B2: Heteroscedasticity in the Transitory and Permanent Component of Growth. This figure plots residuals against fitted values from AR(1) models for our measure of the transitory process (Eq. (B1), left graph) and for EGV (σ, right graph). All variables are standardized.

Table B2: The Cyclicality of the Transitory Process

	Transitory Process				
	(1)	(2)	(3)	(4)	(5)
Constant	0.193***	0.187***	0.244***	0.194***	-0.445***
	(15.85)	(16.12)	(7.76)	(19.53)	(-7.45)
NBER Recession	-0.063*				
	(-1.93)				
Detrended Labor Share		-0.629**			
		(-2.03)			
Default Spread			-0.056**		
			(-2.08)		
NFCI				-0.043***	
				(-5.80)	
ln(P/D)					0.174***
					(11.40)
Observations	205	205	205	196	205
Adj. R^2	0.06	0.05	0.06	0.26	0.54

Note. This table reports estimates from contemporaneous regressions at quarterly frequency of the transitory process in Eq. (B1) on selected measures of macroeconomic (NBER recessions and the detrended labor share) and financial conditions (the default spread, the NFCI, and the logarithm of the price-dividend ratio of the CRSP value-weighted index) over the period 1968-2019. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

C Other Results

Table C1: Definition of Main Variables

Variable	Sources	Definition
Realized and Expected Growth (Volatility)		
g	FRED	Actual quarterly growth rates for real GDP, PCE, IP, and CP. For consistency with SPF mean growth forecest about CP, the latter time series is adjusted for inventory valuation and capital consumption adjustments around 2006.
f	SPF	Quarterly mean growth forecasts for real GDP, PCE, IP, and CP.
σ	SPF	EGV of growth based on mean growth forecasts of real GDP, PCE, IP, and CP for the next quarter and on equations (1)-(2).
σ (AR(MA)(1,1)-(G)ARCH(1,1))	SPF	EGV based on mean growth forecasts of real GDP, PCE, IP, and CP for the next quarter and on a AR(MA)(1,(1))-(G)ARCH((1),1) model for conditional volatility.
$\sigma_{\text{Cond.}}$	SPF, FRED	EGV of GDP growth based on filtered forecasts of GDP, i.e., residuals from a regression of mean growth forecasts of GDP for the next quarter on the (de-trended) labor share of the nonfinancial corporate sector. The econometric approach is the same as in equations (1)-(2).
$\sigma_{\text{Disp.}}$	SPF	The difference between the 75th and the 25th percentile of real GDP growth forecasts for the next quarter.
Stock Market		
ln(P/D)	CRSP, Robert Shiller's Webpage	Natural logarithm of the quarterly price-dividend ratio of the CRSP value-weighted index or of the S&P 500 index. The baseline analysis relies on the CRSP index, except when using data on equity yields from BBK12: because those yields are extracted from options on the S&P 500 index, in that case we use information on the S&P 500 index. To extract information on dividends on the CRSP index, we exploit differences between total and ex-dividend returns as in Eaton and Paye (2017). As in Eaton and Paye (2017), we then compute dividends in a given month as the moving average over the previous year to mitigate seasonal patterns. Available data both for the CRSP and the S&P 500 index are monthly and are converted to quarterly frequency by taking the average of the ratio over the previous three months in each quarter.
Equity Yield Slope (GKK21)	GKK21	Difference between a long-maturity yield ($ey(t,\text{long})$) and the two-year equity yield ($ey(t,\text{short})$), where yields are model-implied measures. The maturity of the long-maturity yield ranges between ten and 100 years. A measure relying on the dividend yield of the CRSP value-weighted index as $ey(t,\text{long})$ is also computed. Available data are monthly and are converted to quarterly frequency by taking the average of the yields over the previous three months in each quarter.
Equity Yield Slope (BBK12)	BBK12	Difference between the S&P 500 dividend yield ($ey(t,\text{long})$) and short-maturity equity yields ($ey(t,\text{short})$), whose maturity ranges between 0.5 and two years. Short-maturity equity yields are extracted from data on dividend prices and current dividends by BBK12 by means of the formula $ey(t,\text{short}) = -\frac{1}{n}\ln\left(\frac{P_{n,t}}{nD_t}\right)$, where $P_{n,t}$ denotes the price of dividends up to maturity n at time t, D_t is the current annual dividend at time t, and $n = 0.5,\ 1,\ 2$ years. Note that, differently from the standard formula to extract equity yields from dividend futures (e.g., Bansal et al., 2021), $P_{n,t}$ is obtained from option prices and needs to be divided by n. Indeed, $P_{n,t}$ in BBK12 is the price of a claim on *all* dividends paid up to maturity n (i.e., not on the single dividend paid at date n). Available data are monthly and are converted to quarterly frequency by taking the average of the yields over the previous three months in each quarter.

(Continued)

Table C1: – *Continued*

Equity Term Premium	CRSP, GKK20, FRED	Difference between the CRSP value-weighted index logarithmic real return (long-maturity claim) and the logarithmic real return on the two-year dividend strip based on the corresponding model-implied equity yield by GKK20. CRSP index returns are monthly and are converted to quarterly frequency by summing them over the previous three months in each quarter. The quarterly return on two-year dividend strip at time t is computed as $-1.75ey_{1.75,t} + 2ey_{2,t-0.25} + \ln\left(\frac{D_t}{D_{t-0.25}}\right)$, where the 1.75-year equity yield is obtained by interpolating the one- and the two-year yields. Finally, the one-quarter to ten-year ahead cumulative equity term premia are computed.
Value, Growth, Value – Growth	Kenneth French's Website	Real dividend growth rates and returns on the value (growth) returns correspond to the top (bottom) decile of stocks sorted on the book-to-market ratio and their difference (value–growth), where portfolio construction follows Fama and French (1992). Portfolio dividend growth rates and returns are monthly and are converted to quarterly frequency by summing them over the previous three months in each quarter. Finally, the one to ten-year ahead cumulative dividend growth rates and returns for each of the three portfolio strategies are computed. To extract information on dividend growth rates on the top and bottom decile portfolio of stocks sorted on the book-to-market ratio, we exploit differences between total and ex-dividend returns as in Eaton and Paye (2017). As in Eaton and Paye (2017), we then compute dividends in a given month as the moving average over the previous year to mitigate seasonal patterns. Available data both for the CRSP and the S&P 500 index are monthly and are converted to quarterly frequency by taking the average of the ratio over the previous three months in each quarter.
Other macrofinance variables		
Inflation	FRED	Quarterly logarithmic inflation rate computed from the seasonally adjusted Consumer Price Index for All Urban Consumers (CPI). The conversion of other variables in real terms is based on this CPI measure.
NBER Recession	FRED	Indicator equal to one if at least one month in a given quarter is classified as a recession by the NBER. Each observation corresponds to the first month of the quarter.
Detrended Labor Share	FRED	Resisuals from a regression of the labor share for employees of the nonfinancial corporate sector on a time trend.
CFNAI	FRED	The three-month moving average of the CFNAI as of the end of the first month of each quarter.
NFCI	SPF	Chicago Fed's NFCI at quarterly frequency.
Default Spread	FRED	The difference between the yield to maturity of Aaa- and Baa-rated corporate bonds. Available data are monthly and are converted to quarterly frequency by taking the average of the spread over the previous three months in each quarter.

Table C2: Goodness of Fit of Alternative EGV Specifications

	RMSE	Correlation with baseline
Specification	(1)	(2)
Panel A		
σ – Eq. (1)-(2)	0.0021	1.0000
σ (AR(1)-ARCH(1))	0.0029	0.6451
σ (AR(1)-GARCH(1,1))	0.0030	0.8904
σ (ARMA(1,1)-ARCH(1))	0.0030	0.6568
σ (ARMA(1,1)-GARCH(1,1))	0.0030	0.8841
Panel B		
$\sigma_{\text{Cond.}}$ – Eq. (1)-(2)	0.0020	0.9966
$\sigma_{\text{Cond.}}$ (AR(1)-ARCH(1))	0.0028	0.6463
$\sigma_{\text{Cond.}}$ (AR(1)-GARCH(1,1))	0.0029	0.8788
$\sigma_{\text{Cond.}}$ (ARMA(1,1)-ARCH(1))	0.0029	0.6590
$\sigma_{\text{Cond.}}$ (ARMA(1,1)-GARCH(1,1))	0.0030	0.8729
Panel C		
σ_{PCE} – Eq. (1)-(2)	0.0015	0.6811
σ_{PCE} (AR(1)-ARCH(1))	0.0022	0.4156
σ_{PCE} (AR(1)-GARCH(1,1))	0.0023	0.6440
σ_{PCE} (ARMA(1,1)-ARCH(1))	0.0021	0.4375
σ_{PCE} (ARMA(1,1)-GARCH(1,1))	0.0023	0.6731
Panel D		
σ_{IP} – Eq. (1)-(2)	0.0051	0.8905
σ_{IP} (AR(1)-ARCH(1))	0.0069	0.6170
σ_{IP} (AR(1)-GARCH(1,1))	0.0068	0.8160
σ_{IP} (ARMA(1,1)-ARCH(1))	0.0071	0.6261
σ_{IP} (ARMA(1,1)-GARCH(1,1))	0.0068	0.8127
Panel E		
σ_{CP} – Eq. (1)-(2)	0.0129	0.6910
σ_{CP} (AR(1)-ARCH(1))	0.0185	0.4230
σ_{CP} (AR(1)-GARCH(1,1))	0.0191	0.6760
σ_{CP} (ARMA(1,1)-ARCH(1))	0.0180	0.4523
σ_{CP} (ARMA(1,1)-GARCH(1,1))	0.0187	0.6672

Note. This table, in column 1, reports the root-mean-square error (RMSE)

$$\text{RMSE} = \left((1/T)\sum_{t=0}^{T-1}\left(|v_{t+1}| - \sigma_{t,t+1}\right)^2\right)^{1/2}$$

from several specifications at quarterly frequency of conditional and volatility models as well as the correlation of the EGV measure they generate with the baseline EGV measure (i.e., based on Eq. (1)-(2)) in column 2 for the period 1968-2019. In Panel A, the EGV measures are estimated from GDP growth forecasts. In Panel B, the EGV measures are estimated after conditioning growth forecasts on the (de-trended) labor share of the corporate sector. In Panel C, D, and E, the EGV measures are estimated from PCE, IP, and CP growth forecasts, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C3: Equity Yield Slope, EGV, and Expected Growth Tails

Panel A

	*n*th quantile of g					
	(1) 5th	(2) 10th	(3) 25th	(4) 75th	(5) 90th	(6) 95th
Constant	-0.004**	-0.000	0.003***	0.008***	0.011***	0.013***
	(-2.46)	(-0.18)	(6.06)	(13.83)	(17.16)	(18.38)
Equity Yield Slope	0.048**	0.050***	0.029***	0.002	-0.017**	-0.023***
(MKT-2Y, GKK21)	(2.15)	(3.31)	(5.72)	(0.59)	(-2.19)	(-2.79)
Observations	180	180	180	180	180	180
Pseudo R^2	0.23	0.22	0.11	0.00	0.05	0.09

Panel B

	*n*th quantile of g					
	(1) 5th	(2) 10th	(3) 25th	(4) 75th	(5) 90th	(6) 95th
Constant	0.007***	0.007***	0.007***	0.007***	0.007***	0.006***
	(14.67)	(16.45)	(14.55)	(16.61)	(13.29)	(7.25)
σ	-3.198***	-3.044***	-1.743***	0.810***	1.675***	2.694***
	(-10.65)	(-10.35)	(-4.08)	(3.18)	(3.35)	(7.73)
Observations	205	205	205	205	205	205
Pseudo R^2	0.47	0.39	0.15	0.07	0.15	0.16

Note. This table reports estimates from contemporaneous quantile regressions at quarterly frequency of expected GDP growth (g) on either the equity yield slope in Panel A or EGV (σ) in Panel B. The dependent variable is the nth quantile of the mean GDP forecast from SPF (g),. The relevant quantile is indicated at the top of each column. Panel A uses the equity yield slope computed as the difference between the dividend yield of the CRSP value-weighted index and the model-implied two-year equity yield of GKK21 for the period 1974-2019. Panel B uses our baseline EGV for the period 1968-2019. The t-statistics are reported in parentheses and are based on robust standard errors. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C4: Alternative Equity Term Premium Measures and EGV

	10Y-Ahead Equity Term Premium		
	MKT-2Y (GKK21)	100Y-2Y (GKK21)	MKT-ST (BBK12)
	(1)	(2)	(3)
Constant	0.640*	0.120	-5.231***
	(1.87)	(0.26)	(-9.74)
σ	416.383***	273.158	1485.352***
	(3.47)	(1.65)	(4.40)
Observations	137	137	11
Adj. R^2	0.21	0.08	0.61
No. lags (Newey-West)	3	3	1

Note. This table reports estimates from regressions at quarterly frequency of alternative measure of the equity term premium on EGV (σ). The equity term premium is the ten-year ahead cumulative long-minus-short equity return. In column 1, the long leg is the CRSP index return and the short one is the two-year dividend strip return. In column 2, the long leg is the 100-year dividend strip return and the short one is the two-year dividend strip return. The strip returns are obtained from data by GKK21 on model-implied equity yields for the period 1974-2019. In column 3, the long leg is the S&P 500 index return and the short one is the return on the BBK12 long strategy in the short-term asset over the period 1996-2009. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C5: The Cyclicality of the Equity Term Structure

Panel A

	Equity Yield Slope (MKT-2Y, GKK21)				
	(1)	(2)	(3)	(4)	(5)
Constant	0.026***	0.017*	0.126***	0.012	-0.524***
	(2.65)	(1.77)	(7.29)	(1.46)	(-19.58)
NBER Recession	-0.076***				
	(-2.73)				
Detrended Labor Share		-0.262			
		(-0.82)			
Default Spread			-0.100***		
			(-9.01)		
NFCI				-0.047***	
				(-8.12)	
ln(P/D)					0.148***
					(18.49)
Observations	180	180	180	180	180
Adj. R^2	0.11	0.01	0.38	0.33	0.70

Panel B

	Equity Yield Slope (MKT-2Y, BBK12)				
	(1)	(2)	(3)	(4)	(5)
Constant	0.012	0.011	0.049*	-0.016	0.562
	(0.84)	(0.85)	(1.73)	(-0.98)	(1.22)
NBER Recession	-0.090***				
	(-2.83)				
Detrended Labor Sshare		-1.339***			
		(-2.88)			
Default Spread			-0.052***		
			(-3.41)		
NFCI				-0.048***	
				(-3.14)	
ln(P/D)					-0.139
					(-1.23)
Observations	55	55	55	55	55
Adj. R^2	0.17	0.31	0.09	0.15	0.14

Panel C					
	10Y-Ahead Equity Term Premium (MKT-2Y, GKK21)				
	(1)	(2)	(3)	(4)	(5)
Constant	1.567***	1.852***	-0.308	1.689***	12.767***
	(6.21)	(7.16)	(-0.71)	(8.21)	(22.89)
NBER Recession	0.825				
	(1.44)				
Detrended Labor Share		-8.900			
		(-0.81)			
Default Spread			1.819***		
			(6.48)		
NFCI				0.813***	
				(7.72)	
ln(P/D)					-3.090***
					(-19.71)
Observations	137	137	137	137	137
Adj. R^2	0.03	0.01	0.27	0.29	0.90

Note. This table reports estimates from regressions at quarterly frequency of the equity yield slope and the equity term premium on several measures proxying for the state of the economy. Such measures comprise an indicator for NBER recessions, the detrended labor share, the default spread, the NFCI, and the logarithm of the price-dividend ratio of the CRSP value-weighted index. In Panel A, the dependent variable is a measure of the equity yield slope based on data by GKK21 for the period 1974-2019. In Panel B, the dependent variable is a measure of the equity yield slope based on data by BBK12 for the period 1996-2009. In Panel C, the dependent variable is the ten-year ahead equity term premium. The maturities of the long and short legs considered to compute the equity yield slope and the equity term premium are indicated in table headers. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C6: Equity Term Structure, EGV, and Other Factors

Panel A

	Equity Yield Slope (MKT-2Y, GKK21)											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.065***	0.064***	0.065***	0.072***	0.072***	0.072***	0.064***	0.065***	0.064***	0.065***	0.065***	0.066***
	(5.20)	(5.45)	(5.05)	(5.72)	(5.56)	(5.41)	(5.36)	(5.28)	(5.12)	(5.25)	(5.23)	(5.12)
σ	-21.809***	-21.067***	-21.589***	-22.507***	-22.507***	-22.507***	-21.233***	-21.659***	-21.280***	-21.666***	-21.922***	-22.326***
	(-4.98)	(-5.67)	(-5.24)	(-3.84)	(-3.82)	(-3.69)	(-5.42)	(-5.04)	(-4.91)	(-5.49)	(-4.98)	(-4.92)
$f \perp \sigma$		2.941**										
		(2.08)										
$\sigma_{g-f} \perp \sigma$			-3.478									
			(-1.04)									
$f_{\text{PCE}} \perp \sigma$				3.709								
				(1.36)								
$\sigma_{\text{PCE}} \perp \sigma$					-20.352***							
					(-3.50)							
$\sigma_{g_{\text{PCE}}-f_{\text{PCE}}} \perp \sigma$						-0.914						
						(-0.15)						
$f_{\text{IP}} \perp \sigma$							1.200*					
							(1.81)					
$\sigma_{\text{IP}} \perp \sigma$								-2.668				
								(-0.84)				
$\sigma_{g_{\text{IP}}-f_{\text{IP}}} \perp \sigma$									-4.301***			
									(-2.74)			
$f_{\text{CP}} \perp \sigma$										0.425**		
										(2.03)		
$\sigma_{\text{CP}} \perp \sigma$											0.442	
											(0.58)	
$\sigma_{g_{\text{CP}}-f_{\text{CP}}} \perp \sigma$												0.395
												(1.06)
Observations	180	180	180	154	154	154	180	180	180	180	180	180
Adj. R^2	0.25	0.29	0.26	0.20	0.25	0.17	0.28	0.26	0.29	0.28	0.25	0.26

Panel B

	Equity Yield Slope (MKT-2Y, BKK12)											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.036**	0.026	0.047***	0.033*	0.034*	0.036*	0.026	0.037**	0.029*	0.032**	0.026	0.031*
	(2.12)	(1.53)	(3.82)	(1.95)	(1.79)	(1.77)	(1.55)	(2.06)	(1.68)	(2.07)	(1.53)	(1.75)
σ	-19.323***	-12.998***	-25.289***	-17.483***	-18.605***	-19.389***	-11.198***	-20.096***	-13.948***	-20.122***	-15.632***	-14.537**
	(-4.19)	(-3.93)	(-7.59)	(-4.00)	(-3.68)	(-3.97)	(-2.92)	(-2.73)	(-2.72)	(-4.93)	(-3.09)	(-2.16)
$f \perp \sigma$		7.617***										
		(2.89)										
$\sigma_{g-f} \perp \sigma$			-21.056***									
			(-3.56)									
$f_{\mathrm{PCE}} \perp \sigma$				3.383								
				(1.26)								
$\sigma_{\mathrm{PCE}} \perp \sigma$					9.610							
					(0.41)							
$\sigma_{g_{\mathrm{PCE}}-f_{\mathrm{PCE}}} \perp \sigma$						-0.331						
						(-0.02)						
$f_{\mathrm{IP}} \perp \sigma$							4.851***					
							(3.17)					
$\sigma_{\mathrm{IP}} \perp \sigma$								-1.730				
								(-0.17)				
$\sigma_{g_{\mathrm{IP}}-f_{\mathrm{IP}}} \perp \sigma$									-4.859***			
									(-2.73)			
$f_{\mathrm{CP}} \perp \sigma$										1.448***		
										(2.81)		
$\sigma_{\mathrm{CP}} \perp \sigma$											2.107***	
											(3.14)	
$\sigma_{g_{\mathrm{CP}}-f_{\mathrm{CP}}} \perp \sigma$												-0.414
												(-0.94)
Observations	55	55	55	55	55	55	55	55	55	55	55	55
Adj. R^2	0.14	0.26	0.29	0.14	0.13	0.12	0.32	0.12	0.18	0.32	0.20	0.13

Panel C

	10Y-Ahead Equity Term Premium (MKT-2Y, GKK21)											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.640*	0.649*	0.630*	0.284	0.149	0.229	0.640*	0.650**	0.634*	0.664*	0.640*	0.722**
	(1.87)	(1.92)	(1.91)	(0.72)	(0.40)	(0.56)	(1.86)	(2.01)	(1.86)	(1.97)	(1.88)	(2.06)
σ	416.383***	412.407***	408.560***	498.512**	524.977**	508.823**	416.193***	403.665***	433.568***	407.713***	415.087***	380.093***
	(3.47)	(3.50)	(4.05)	(2.43)	(2.58)	(2.38)	(3.46)	(3.62)	(3.50)	(3.56)	(3.53)	(3.08)
$f \perp \sigma$		-10.311										
		(-0.32)										
$\sigma_{g-f} \perp \sigma$			160.711**									
			(2.47)									
$f_{\text{PCE}} \perp \sigma$				2.728								
				(0.06)								
$\sigma_{\text{PCE}} \perp \sigma$					381.210***							
					(2.74)							
$\sigma_{g_{\text{PCE}}-f_{\text{PCE}}} \perp \sigma$						101.203						
						(0.94)						
$f_{\text{IP}} \perp \sigma$							-0.344					
							(-0.02)					
$\sigma_{\text{IP}} \perp \sigma$								98.441				
								(1.47)				
$\sigma_{g_{\text{IP}}-f_{\text{IP}}} \perp \sigma$									103.991**			
									(2.01)			
$f_{\text{CP}} \perp \sigma$										-5.458		
										(-1.14)		
$\sigma_{\text{CP}} \perp \sigma$											1.944	
											(0.12)	
$\sigma_{g_{\text{CP}}-f_{\text{CP}}} \perp \sigma$												-15.422
												(-1.15)
Observations	137	137	137	110	110	110	137	137	137	137	137	137
Adj. R^2	0.21	0.20	0.25	0.15	0.23	0.16	0.20	0.23	0.25	0.22	0.20	0.23

Note. This table reports estimates from regressions at quarterly frequency of the equity yield slope and the equity term premium on EGV and other orthogonal measures of growth. In Panel A, the dependent variable is a measure of the equity yield slope based on data by GKK21 for the period 1974-2019. In Panel B, the dependent variable is a measure of the equity yield slope based on data by BBK12 for the period 1996-2009. In Panel C, the dependent variable is the ten-year ahead equity term premium. The maturities of the long and short legs considered to compute the equity yield slope and the equity term premium are indicated in table headers. In each panel, column 1 regresses the dependent variable on baseline EGV (σ) alone. Each of columns 2-12 augments such a specification with a measure of growth orthogonal to EGV. The considered measures—all orthogonalized with respect to baseline EGV—are the GDP growth forecast ($f \perp \sigma$), the conditional volatility of residual GDP growth ($\sigma_{g-f} \perp \sigma$), the EGV of PCE ($f_{\text{PCE}} \perp \sigma$), the PCE growth forecast ($\sigma_{\text{PCE}} \perp \sigma$), the conditional volatility of residual PCE growth ($\sigma_{g_{\text{PCE}}-f_{\text{PCE}}} \perp \sigma$), EGV of IP ($f_{\text{IP}} \perp \sigma$), the IP growth forecast ($\sigma_{\text{IP}} \perp \sigma$), the conditional volatility of residual IP growth ($\sigma_{g_{\text{IP}}-f_{\text{IP}}} \perp \sigma$), the EGV of CP growth ($f_{\text{CP}} \perp \sigma$), the CP growth forecast ($\sigma_{\text{CP}} \perp \sigma$), and the conditional volatility of residual CP growth ($\sigma_{g_{\text{CP}}-f_{\text{CP}}} \perp \sigma$). The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C7: Equity Term Structure, EGV, and the Covariance of Expected Growth Innovations with Residual Growth

	Equity Yield Slope				10Y-Ahead Equity Term Premium	
	MKT-2Y (GKK21)		MKT-2Y (BBK12)		MKT-2Y (GKK21)	
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	0.717***	0.913***	0.390	0.510**	0.520**	0.416*
	(3.46)	(5.37)	(1.36)	(2.56)	(2.12)	(1.84)
σ_{RW}	-0.374***		-0.294		0.442***	
	(-3.29)		(-1.45)		(3.51)	
$\text{Cov}_{RW}(v, g-f) \perp \sigma_{RW}$	-0.199*		-0.508*		0.083	
	(-1.67)		(-2.00)		(0.65)	
σ		-0.549***		-0.430***		0.517***
		(-5.60)		(-3.34)		(4.01)
$\text{Cov}_{RW}(v, g-f) \perp \sigma$		-0.068		-0.380		0.050
		(-0.68)		(-1.58)		(0.43)
Observations	180	180	55	55	137	137
Adj. R^2	0.12	0.25	0.18	0.20	0.15	0.21

Note. This table reports estimates from regressions at quarterly frequency of the equity yield slope and the equity term premium on EGV and other measures of growth. In columns 1-2, the dependent variable is a measure of the equity yield slope based on data by GKK21 for the period 1974-2019. In columns 3-4, the dependent variable is a measure of the equity yield slope based on data by BBK12 for the period 1996-2009. In columns 5-6, the dependent variable is the ten-year ahead equity term premium. The maturities of the long and short legs considered to compute the equity yield slope and the equity term premium are indicated in the table header. Odd columns regress the dependent variable on: (i) σ_{RW}, namely the standard deviation of residuals v from an AR(1) model for GDP growth forecast as in Eq. (1), computed over a rolling window of five observations, and (ii) $\text{Cov}_{RW}(v, g-f) \perp \sigma_{RW}$, namely the covariance between v and residual GDP growth, computed over a rolling window of five observations and orthogonalized with respect to σ_{RW}. Even columns regress the dependent variable on: (i) σ, namely baseline EGV, and (ii) $\text{Cov}_{RW}(v, g-f) \perp \sigma$, namely the covariance between v and residual GDP growth, computed over a rolling window of five observations and orthogonalized with respect to σ. Both dependent and explanatory variables are standardized to favor readibility. The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C8: Alternative EGV Measures and Asset Prices

	Price Dynamics		Equity Yield Slope (MKT-2Y)		Equity Term Premium (MKT-2Y)	10Y-Ahead Value-Growth Premium	
	(1) ln(P/D)	(2) $\sigma_{\ln(P/D)}$	(3) BBK12	(4) GKK21	(5) 10Y-Ahead (GKK21)	(6) Div. growth	(7) Return
σ (AR(1)-ARCH(1))	-102.657***	4.274**	-24.293***	-29.603***	325.252***	108.009*	66.518
	(-4.367)	(2.315)	(-4.247)	(-3.59)5	(2.864)	(1.667)	(1.273)
σ (AR(1)-GARCH(1,1))	-100.499***	3.860***	-17.450***	-14.730***	380.074***	138.191**	85.912**
	(-5.984)	(3.321)	(-4.954)	(-3.168)	(3.898)	(2.438)	(2.098)
σ (ARMA(1,1)-ARCH(1))	-89.392***	3.855**	-20.024***	-24.713***	274.454***	93.616*	55.768
	(-4.687)	(2.562)	(-4.150)	(-3.407)	(2.841)	(1.664)	(1.192)
σ (ARMA(1,1)-GARCH(1,1))	-98.950***	3.993***	-16.470***	-14.720***	359.289***	135.407**	84.247**
	(-6.166)	(3.589)	(-4.678)	(-3.060)	(3.780)	(2.482)	(2.079)
$\sigma_{\text{Disp.}}$	-112.004***	1.713	-19.739***	-35.546***	397.434***	106.136**	105.808***
	(-9.981)	(1.292)	(-7.245)	(-4.616)	(6.098)	(2.289)	(3.321)
$\sigma_{\text{Cond.}}$	-118.676***	4.777***	-21.908***	-19.230***	424.917***	120.845*	81.090
	(-5.841)	(3.302)	(-4.877)	(-3.976)	(3.451)	(1.736)	(1.490)
σ_{PCE}	-123.709***	4.736***	-26.527***	-20.684**	519.726***	-201.925**	-145.289**
	(-3.706)	(3.015)	(-4.661)	(-2.257)	(3.252)	-2.527	-2.152
σ_{IP}	-49.803***	1.654**	-8.084***	-9.677***	158.241***	38.170	31.709*
	(-7.841)	(2.212)	(-6.428)	(-2.952)	(4.900)	(1.515)	(1.747)
σ_{CP}	-14.153***	0.638***	-1.996**	0.880	40.832**	4.400	-1.332
	(-3.211)	(2.996)	(-2.498)	(0.875)	(2.370)	(0.411)	(-0.156)

Note. This table reports slope estimates from regressions at quarterly frequency of several of asset pricing quantities on alternatives measures of the EGV of GDP (and of other macroeconomic aggregates). The dependent variables are the logarithm of the price-dividend ratio of the CRSP value-weighted index, its conditional volatility computed following Eq. (2), the equity yield slope based either on GKK21 or BBK12 data, the ten-year ahead cumulative equity term premium based on GKK21 data, and the ten-year ahead cumulative dividend growth and return on value (firms belonging to the top decide of the book-to-market ratio), growth (firms belonging to the bottom decide of the book-to-market ratio) and value-minus-growth portfolios. The EGV measures include AR(MA)(1,1)-(G)ARCH(1,1) specifications, the cross-sectional dispersion of GDP growth forecasts ($\sigma_{\text{Disp.}}$) as well EGV based on Eq. (1)-(2) applied to growth forecasts for: GDP after removing the cyclical component as captured by the detrended labor share of the corporate sector ($\sigma_{\text{Cond.}}$), PCE (σ_{PCE}), IP (σ_{IP}), and CP (σ_{CP}). The t-statistics are reported in parentheses and are based on Newey-West standard errors with a number of lags equal to the integer part of $T^{0.25}$, where T is the number of observations. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively. Detailed variable definitions are provided in Appendix Table C1.

Table C9: Alternative Calibration: Standard Moments

Moment	Data	Model				
		2.5%	5%	50%	95%	97.5%
Avg. consumption growth	0.021	-0.001	0.004	0.026	0.043	0.046
Std. consumption growth	0.018	0.025	0.026	0.031	0.040	0.042
Avg. payout growth	0.030	-0.002	0.003	0.026	0.044	0.047
Std. payout growth						
- *Corporate profits*	0.148	0.151	0.156	0.181	0.207	0.212
- *Dividends plus net repurchases*	0.266					
20-year std. payout growth						
- *Corporate profits*	0.083	0.019	0.028	0.101	0.216	0.240
- *Dividends plus net repurchases*	0.118					
Std. log payout-consumption ratio	0.228	0.163	0.172	0.230	0.309	0.327
Corr. consumption and payout exp. growth	0.822	0.246	0.3122	0.660	0.849	0.872
Corr. payout exp. growth and exp. growth volatility	-0.331	-0.737	-0.697	-0.363	0.079	0.157
Avg. risk-free rate	0.007	-0.034	-0.028	0.001	0.028	0.033
Std. risk-free rate	0.025	0.026	0.027	0.038	0.053	0.057
Avg. excess equity return	0.068	0.042	0.047	0.074	0.100	0.106
Std. excess equity return	0.175	0.119	0.123	0.142	0.162	0.166
Avg. Sharpe ratio	0.388	0.292	0.326	0.519	0.725	0.772
Avg. log price-dividend ratio	3.435	2.884	2.902	2.994	3.073	3.087
Std. log price-dividend ratio	0.443	0.071	0.075	0.104	0.150	0.161
Avg. excess high-minus-low return	0.035	0.011	0.018	0.058	0.105	0.115
Std. excess high-minus-low return	0.129	0.112	0.116	0.135	0.156	0.160

Note. This table reports moment statistics from both data and model simulations. Model-implied statistics are moment quantiles from short-sample (75 years) simulations. The model is simulated at monthly frequency. Statistics are yearly moments if not stated otherwise. Consumption and payout (corporate profits) data are from the National Income and Product Accounts (NIPA). Returns are from K. French webpage. The price-dividend ratio is from R. Shiller webpage. Dividends plus net repurchases are from Belo et al. (2015).

D Model Extensions

Heteroscedastic Transitory Risk: Model Derivation

Setup

In this model extension, we consider a less simplistic assumption about volatility in fundamentals. Whereas the baseline model assumes that the transitory shock z_t is homoscedastic, we here relax this assumption by assuming that z_t is heteroscedastic. In particular, a small fraction of the conditional variance of z_t is driven by the conditional variance of the permanent shock x_t, consistent with empirical evidence (see Appendix B). Specifically, in our extended framework, the permanent and the transitory components follow:

$$dx_t = \lambda_x(\bar{x} - x_t)dt + \sigma_x\sqrt{x_t}\, dB_{x,t}, \tag{D1}$$

$$dz_t = -\lambda_z z_t dt + \sqrt{\sigma_{z_0}^2 + \sigma_{zx}^2\, x_t}\, dB_{z,t}. \tag{D2}$$

The Brownian shocks $B_{x,t}$ and $B_{z,t}$ are assumed to be independent. Heteroscedasticity of z_t is governed by the loading σ_{zx} and setting σ_{zx} to zero yields the baseline setup. Section V.B shows that the model mechanism and equity term structure dynamics of the baseline model are robust to the specification of Eq. (D1)-(D2), as long as $\sigma_{zx} \times \mathbb{E}[x_t]$ represents a modest fraction of the average conditional variance of z_t, in accord with the data.

Affine Notation

The vector $X_t = (y_t, x_t, z_t, w_,t)^\intercal$ collects the two state variables of our model x_t and z_t as well as accumulated expected growth $y_t = \mu t + \int_0^t(\bar{x} - x_s)ds$ and the component of cross-sectional payouts $w_t = \varphi\int_0^t(\bar{x} - x_s)ds + \sigma_\varphi B_{\varphi,t}$. The vector belongs to the affine class and has dynamics:

$$\begin{aligned} dX_t &= \mu(X_t)dt + \Sigma(X_t)d\mathcal{B}_t, \\ \mu(X_t) &= \mathcal{M} + \mathcal{K}X_t, \\ \Sigma(X_t)\Sigma(X_t)^\intercal &= h + \sum\nolimits_{i\in\{y,x,z,w\}} H_i X_{i,t}, \end{aligned}$$

with Brownian motions $\mathcal{B}_t = (B_{y,t}, B_{x,t}, B_{z,t}, B_{w,t})^\intercal$ and the following coefficients:

$$\mathcal{M} = \begin{pmatrix} \mu + \bar{x} \\ \lambda_x\bar{x} \\ 0 \\ \varphi\bar{x} \end{pmatrix}, \qquad \mathcal{K} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -\lambda_x & 0 & 0 \\ 0 & 0 & -\lambda_z & 0 \\ 0 & -\varphi & 0 & 0 \end{pmatrix}, \qquad h = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{z0}^2 & 0 \\ 0 & 0 & 0 & \sigma_\varphi^2 \end{pmatrix},$$

$$H_y = H_z = H_w = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad H_x = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 & 0 \\ 0 & 0 & \sigma_{zx}^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The following selection vectors allow to recover consumption, aggregate payouts, and cross-sectional payouts:

$$
\begin{aligned}
v_C &= (1,0,1,0)^\intercal \quad &\Rightarrow \quad & v_C^\intercal X_t = \log C_t, \\
v_D &= (1,0,\phi,0)^\intercal \quad &\Rightarrow \quad & v_D^\intercal X_t = \log D_t, \\
v_\varphi &= (1,0,\phi,1)^\intercal \quad &\Rightarrow \quad & v_\varphi^\intercal X_t = \log D_t^\varphi.
\end{aligned}
$$

Moment Generating Function

The following conditional expectation allows to compute the moment generating function for the logarithm of C, D ,and D^φ at any horizon τ:

$$
\mathbb{E}_t[\exp(\mathbf{u}^\intercal X_{t+\tau})] = \exp(\bar{b}_0(\tau) + \bar{b}(\tau)^\intercal X_t). \tag{D3}
$$

Following Duffie et al. (2000), the functions $\bar{b}_0(\tau)$ and $\bar{b}(\tau) = \left(\bar{b}_y(\tau), \bar{b}_x(\tau), \bar{b}_z(\tau), \bar{b}_w(\tau)\right)^\intercal$ solve the following system of ODE's:

$$
\begin{aligned}
\bar{b}_0'(\tau) &= \mathcal{M}^\intercal \bar{b}(\tau) + \frac{1}{2}\bar{b}(\tau)^\intercal h \bar{b}(\tau), \\
\bar{b}'(\tau) &= \mathcal{K}^\intercal \bar{b}(\tau) + \frac{1}{2}\bar{b}(\tau)^\intercal H \bar{b}(\tau).
\end{aligned}
$$

By setting the initial conditions $\bar{b}_0(0) = 0$ and either $\bar{b}(0) = uv_C$, $\bar{b}(0) = uv_D$, or $\bar{b}(0) = uv_\varphi$, Eq. (D3) computes the time-t conditional expectation of either $C_{t+\tau}, D_{t+\tau}$, or $D_{t+\tau}^\varphi$ with power u respectively. These expectations can be used to build the term structure of growth rates volatility.

Equity and State-Price Density

We follow Eraker and Shaliastovich (2008) and the state-price density based on recursive preferences of Epstein and Zin (1989) type. To do so, we use the Campbell and Shiller (1988) approximation to log-linearize the return R_t on the wealth of market participants (that in our economy corresponds to the equity market and pays-out D_t):

$$
d\log R_t = k_0 dt + k_1 dpd_t - (1-k_1) pd_t dt + d\log D_t, \tag{D4}
$$

Where k_0 and k_1 are endogenous constants to be determined. We conjecture that the log price-payout ratio (which is also the log wealth-consumption ratio of market participants) is an affine function of X_t: $pd_t = A_0 + A^\intercal X_t$. We use Eq. (D4) to rewrite the state-price density dynamics as follows:

$$
\begin{aligned}
d\log M_t &= \theta \log \beta dt - \frac{\theta}{\psi} d\log D_t - (1-\theta) d\log R_t \\
&= \left(\theta \log\beta - (\theta-1)\log k_1 + (\theta-1)(k_1-1) A^\intercal (X_t - \mu_X)\right) dt - \lambda^\intercal dX_t,
\end{aligned} \tag{D5}
$$

where $\lambda = \gamma v_D + (1-\theta) k_1 A$ and $\mu_X = (0, \bar{x}, 0, 0, 0)^\intercal$. Then, the Euler equation can be written as:

$$1 = \mathbb{E}_t \left[\frac{M_{t+\tau}}{M_t} e^{\int_0^\tau d \log R_{t+s}} \right], \qquad \forall \tau.$$

Since the term in the conditional expectation has to be a martingale, we apply Itô's lemma to compute its drift that we set equal to zero:

$$0 = \theta \log \beta + \chi^\intercal (\mathcal{M} + \mathcal{K} X_t) + \theta k_0 - \theta(1-k_1)(A_0 + A^\intercal X_t) + \frac{1}{2} \chi \Sigma(X_t)^\intercal \Sigma(X_t) \chi^\intercal X_t, \quad \text{(D6)}$$

where $\chi = \theta \left(\left(1 - \frac{1}{\psi} \right) v_D + k_1 A \right)$. Since Eq. (D6) holds for all X_t and we set the coefficients on X_t and the residual constant equal to zero. The endogenous coefficients k_1, A_0 and $A = (A_y, A_x, A_z, A_w)^\intercal$ are obtained by solving the following system:

$$\begin{aligned}
0 &= \mathcal{K}^\intercal \chi - \theta (1-k_1) A + \frac{1}{2} \chi^\intercal H \chi, \\
0 &= \theta (\log \beta + k_0 - (1-k_1) A_0) + \mathcal{M}^\intercal \chi + \frac{1}{2} \chi^\intercal h \chi, \\
\theta \log k_1 &= \theta (\log \beta + (1-k_1) A^\intercal \mu_X) + \mathcal{M}^\intercal \chi + \frac{1}{2} \chi^\intercal h \chi.
\end{aligned}$$

The solution coefficients should be inserted into Eq. (D5) to obtain the equilibrium state price density.[25] The equity price is then given by $P_t = D_t \exp(A_0 + A^\intercal X_t)$, where $A_y = A_w = 0$. Applying Itô's Lemma to Eq. (D5) yields:

$$\begin{aligned}
\frac{dM_t}{M_t} &= (\theta \log \beta - (\theta - 1) \log k_1 + (\theta - 1)(k_1 - 1) A^\intercal (X_t - \mu_X) + \mu(X_t)^\intercal \lambda) \, dt \\
&\quad + \frac{1}{2} \lambda^\intercal \Sigma(X_t) \lambda dt - \lambda^\intercal \Sigma(X_t) d\mathcal{B}_t \\
&= -(r_0 + \bar{r}^\intercal X_t) \, dt - \lambda^\intercal \Sigma(X_t) d\mathcal{B}_t,
\end{aligned} \quad \text{(D7)}$$

where the coefficients r_0 and $\bar{r} = (r_y, r_x, r_z, r_w)^\intercal$ are:

$$\begin{aligned}
r_0 &= -\theta \log \beta + (\theta - 1)(\log k_1 + (k_1 - 1) A^\intercal \mu_X) + \mathcal{M}^\intercal \lambda - \frac{1}{2} \lambda^\intercal h \lambda, \\
\bar{r} &= (1-\theta)(k_1 - 1) A + \mathcal{K}^\intercal \lambda - \frac{1}{2} \lambda^\intercal H \lambda.
\end{aligned}$$

Therefore, the risk-free rate is given by $r_t = r_0 + \bar{r}^\intercal X_t$ and the vector of risk prices is given by $\Omega(X_t) = (\Omega_y, \Omega_x, \Omega_z, \Omega_w)^\intercal = \Sigma(X_t)^\intercal \lambda$, where it turns out that $r_y = r_w = 0$ and $\Omega_y = \Omega_w = 0$. Consequently, the risk premium on equity is equal to $RP(x_t) = ((A + v_D)^\intercal \Sigma(X_t)) \Omega(X_t)$, which is an affine function of x_t only.

[25] Note that the above system of equations could yields multiple solutions. Tauchen (2011) proposes to select the root which ensures the non-explosiveness of the system. Alternatively, one could select an economically reasonable solution.

Term Structures

Following Duffie et al. (2000), the risk-neutral dynamics of X_t are given by:

$$\begin{aligned}
dX_t &= \left(\mathcal{M}^{\mathcal{Q}} + \mathcal{K}^{\mathcal{Q}} X_t\right) dt + \Sigma(X_t) d\mathcal{B}_t^{\mathcal{Q}}, \\
\mathcal{M}^{\mathcal{Q}} &= \mathcal{M} - h\lambda, \\
\mathcal{K}^{\mathcal{Q}} &= \mathcal{K} - H\lambda, \\
d\mathcal{B}_t^{\mathcal{Q}} &= d\mathcal{B}_t + \Sigma(X_t)^\intercal \lambda \, dt.
\end{aligned}$$

Then, we can compute the discounted value of several payouts, such as $D_{t+\tau}, D^{\varphi}_{t+\tau}$ and the unitary payout of a risk-less bond:

$$\mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t}\exp(v^\intercal X_{t+\tau})\right] = \mathbb{E}^{\mathcal{Q}}\left[\exp(-\int_0^\tau r_{t+s} ds + v^\intercal X_{t+\tau})\right] = \exp(q_0(\tau) + q(\tau)^\intercal X_t),$$

where $v \in \{v_D, v_\varphi, (0,0,0,0)^\intercal\}$. The deterministic function $q_o(\tau)$ and $q(\tau) = (q_y(\tau), q_x(\tau), q_z(\tau), q_w(\tau))$ solve the following system of ODEs:

$$\begin{aligned}
q_0'(\tau) &= -r_0 + \left(\mathcal{M}^{\mathcal{Q}}\right)^\intercal q(\tau) + \frac{1}{2} q(\tau)^\intercal h \, q(\tau), \\
q'(\tau) &= -\bar{r} + \left(\mathcal{K}^{\mathcal{Q}}\right)^\intercal q(\tau) + \frac{1}{2} q(\tau)^\intercal H q(\tau),
\end{aligned}$$

with initial conditions $q_0(0) = 0$ and $q(0) = v$.

Therefore, the risk-less bond price, the strip price of the aggregate payout and the strip price of the cross-sectional payout are given by

$$\begin{aligned}
B_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = (0,0,0,0)^\intercal, \\
P_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = v_D, \\
P^{\varphi}_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^\intercal X_t), \quad \text{with} \quad v = v_\varphi.
\end{aligned}$$

Cross-Sectional Equity

The price of the stock paying out the stream D^{φ}_t can be computed either as the time integral of the corresponding strip price over any maturity or via an exponential affine approximation. Such exponential affine approximation is given by

$$P^{\varphi}_t = D^{\varphi}_t \exp(A^{\varphi}_0 + (A^{\varphi})^\intercal X_t),$$

where the coefficients A^{φ}_0, $A^{\varphi} = (A^{\varphi}_y, A^{\varphi}_x, A^{\varphi}_z, A^{\varphi}_w)^\intercal$, and the endogenous constant k^{φ}_1 solve the following system:

$$\begin{aligned}
0 &= (\theta - 1)(k_1 - 1)A + (k^{\varphi}_1 - 1)A^{\varphi} + \mathcal{K}^\intercal \chi_\varphi + (1/2)\chi_\varphi^\intercal H \chi_\varphi, \\
0 &= \theta \log\beta - (\theta - 1)(\log k_1 + (k_1 - 1)A^\intercal \mu_X) - (\log k^{\varphi}_1 + (k^{\varphi}_1 - 1)(A^{\varphi})^\intercal \mu_X) + \mathcal{M}^\intercal \chi_\varphi \\
&\quad + (1/2)\chi_\varphi^\intercal h \chi_\varphi,
\end{aligned}$$

$$0 = A_0^\varphi + (A^\varphi)^\intercal \mu_X - \log k_1^\varphi + \log(1-k_1^\varphi),$$

where $\chi_\varphi = v_\varphi + k_1^\varphi A^\varphi - \lambda$. It turns out that $A_y^\varphi = A_w^\varphi = 0$. Therefore, the risk premium on the cross-sectional stock is equal to $RP^\varphi(x_t) = ((A^\varphi + v_\varphi)^\intercal \Sigma(X_t))\Omega(X_t)$, which is an affine function of x_t only.

Full Market Participation: Model Derivation

Setup

In this model extension, we consider the case of full market participation. Whereas the baseline model assumes that only shareholders participate on the equity market, here we assume that both workers and shareholders participate. In particular, we make the standard assumption that a representative agent with recursive preferences of Epstein and Zin (1989) type exsists and in equilibrium consumes total resources ($C = W + D$).

Affine Notation

The vector $X_t = (y_t, x_t, z_t, w_,t)^\intercal$ collects the two state variables of our model x_t and z_t as well as accumulated expected growth $y_t = \mu t + \int_0^t (\bar{x} - x_s) ds$ and the component of cross-sectional payouts $w_t = \varphi \int_0^t (\bar{x} - x_s) ds + \sigma_\varphi B_{\varphi,t}$. The vector belongs to the affine class and has dynamics:

$$\begin{aligned}
dX_t &= \mu(X_t)dt + \Sigma(X_t)d\mathcal{B}_t, \\
\mu(X_t) &= \mathcal{M} + \mathcal{K}X_t, \\
\Sigma(X_t)\Sigma(X_t)^\intercal &= h + \sum\nolimits_{i\in\{y,x,z,w\}} H_i X_{i,t},
\end{aligned}$$

with Brownian motions $\mathcal{B}_t = (B_{y,t}, B_{x,t}, B_{z,t}, B_{w,t})^\intercal$ and the following coefficients:

$$\mathcal{M} = \begin{pmatrix} \mu + \bar{x} \\ \lambda_x \bar{x} \\ 0 \\ \varphi \bar{x} \end{pmatrix}, \qquad \mathcal{K} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -\lambda_x & 0 & 0 \\ 0 & 0 & -\lambda_z & 0 \\ 0 & -\varphi & 0 & 0 \end{pmatrix}, \qquad h = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_\varphi^2 \end{pmatrix},$$

$$H_y = H_z = H_w = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad H_x = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The following selection vectors allow to recover consumption, aggregate payouts, and cross-sectional payouts:

$$\begin{aligned}
v_C &= (1,0,1,0)^\intercal &\Rightarrow\quad v_C^\intercal X_t &= \log C_t, \\
v_D &= (1,0,\phi,0)^\intercal &\Rightarrow\quad v_D^\intercal X_t &= \log D_t, \\
v_\varphi &= (1,0,\phi,1)^\intercal &\Rightarrow\quad v_\varphi^\intercal X_t &= \log D_t^\varphi.
\end{aligned}$$

Moment Generating Function

The following conditional expectation allows to compute the moment generating function for the logarithm of C, D ,and D^{φ} at any horizon τ:

$$\mathbb{E}_t[\exp(\mathbf{u}^\intercal X_{t+\tau})] = \exp(\bar{b}_0(\tau) + \bar{b}(\tau)^\intercal X_t). \tag{D8}$$

Following Duffie et al. (2000), the functions $\bar{b}_0(\tau)$ and $\bar{b}(\tau) = \left(\bar{b}_y(\tau), \bar{b}_x(\tau), \bar{b}_z(\tau), \bar{b}_w(\tau)\right)^\intercal$ solve the following system of ODE's:

$$\begin{aligned}
\bar{b}_0'(\tau) =& \mathcal{M}^\intercal \bar{b}(\tau) + \frac{1}{2}\bar{b}(\tau)^\intercal h \bar{b}(\tau), \\
\bar{b}'(\tau) =& \mathcal{K}^\intercal \bar{b}(\tau) + \frac{1}{2}\bar{b}(\tau)^\intercal H \bar{b}(\tau).
\end{aligned}$$

By setting the initial conditions $\bar{b}_0(0) = 0$ and either $\bar{b}(0) = uv_C$, $\bar{b}(0) = uv_D$, or $\bar{b}(0) = uv_{\varphi}$, Eq. (D8) computes the time-t conditional expectation of either $C_{t+\tau}, D_{t+\tau}$, or $D^{\varphi}_{t+\tau}$ with power u respectively. These expectations can be used to build the term structure of growth rates volatility.

Equity and State-Price Density

We follow Eraker and Shaliastovich (2008) and the state-price density based on recursive preferences of Epstein and Zin (1989) type. To do so, we use the Campbell and Shiller (1988) approximation to log-linearize the return $R_{c,t}$ on the wealth of market participants (that in this economy pays-out C_t):

$$d \log R_{c,t} = k_0 dt + k_1 dwc_t - (1 - k_1) wc_t dt + d \log C_t, \tag{D9}$$

Where k_0 and k_1 are endogenous constants to be determined. We conjecture that the log wealth-consumption ratio of the economy is an affine function of X_t: $wc_t = A_0 + A^\intercal X_t$. We use Eq. (D9) to rewrite the state-price density dynamics as follows:

$$\begin{aligned}
d \log M_t =& \theta \log \beta dt - \frac{\theta}{\psi} d \log C_t - (1 - \theta) d \log R_{c,t} \\
=& \left(\theta \log \beta - (\theta - 1) \log k_1 + (\theta - 1)\left(k_1 - 1\right) A^\intercal \left(X_t - \mu_X\right)\right) dt - \lambda^\intercal dX_t,
\end{aligned} \tag{D10}$$

where $\lambda = \gamma v_C + (1 - \theta)\, k_1 A$ and $\mu_X = (0, \bar{x}, 0, 0, 0)^\intercal$. Then, the Euler equation can be written as:

$$1 = \mathbb{E}_t \left[\frac{M_{t+\tau}}{M_t} e^{\int_0^\tau d \log R_{c,t+s}} \right], \qquad \forall \tau.$$

Since the term in the conditional expectation has to be a martingale, we apply Itô's lemma to compute its drift that we set equal to zero:

$$0 = \theta \log \beta + \chi^\intercal \left(\mathcal{M} + \mathcal{K} X_t\right) + \theta k_0 - \theta(1 - k_1)(A_0 + A^\intercal X_t) + \frac{1}{2}\chi \Sigma(X_t)^\intercal \Sigma(X_t) \chi^\intercal X_t, \tag{D11}$$

where $\chi = \theta\left(\left(1-\frac{1}{\psi}\right)v_C + k_1 A\right)$. Since Eq. (D11) holds for all X_t and we set the coefficients on X_t and the residual constant equal to zero. The endogenous coefficients k_1, A_0 and $A = (A_y, A_x, A_z, A_w)^\intercal$ are obtained by solving the following system:

$$
\begin{aligned}
0 &= \mathcal{K}^\intercal \chi - \theta\left(1-k_1\right)A + \frac{1}{2}\chi^\intercal H \chi, \\
0 &= \theta\left(\log\beta + k_0 - \left(1-k_1\right)A_0\right) + \mathcal{M}^\intercal \chi + \frac{1}{2}\chi^\intercal h \chi, \\
\theta \log k_1 &= \theta\left(\log\beta + \left(1-k_1\right)A^\intercal \mu_X\right) + \mathcal{M}^\intercal \chi + \frac{1}{2}\chi^\intercal h \chi.
\end{aligned}
$$

The solution coefficients should be inserted into Eq. (D10) to obtain the equilibrium state price density.[26] The wealth is then given by $\mathcal{W}_t = C_t \exp(A_0 + A^\intercal X_t)$, where $A_y = A_w = 0$. Applying Itô's Lemma to Eq. (D10) yields:

$$
\begin{aligned}
\frac{dM_t}{M_t} &= \left(\theta\log\beta - (\theta-1)\log k_1 + (\theta-1)\left(k_1-1\right)A^\intercal\left(X_t-\mu_X\right) + \mu(X_t)^\intercal\lambda\right)dt \\
&\quad + \frac{1}{2}\lambda^\intercal\Sigma(X_t)\lambda dt - \lambda^\intercal\Sigma(X_t)d\mathcal{B}_t \\
&= -\left(r_0 + \bar{r}^\intercal X_t\right)dt - \lambda^\intercal\Sigma(X_t)d\mathcal{B}_t,
\end{aligned} \tag{D12}
$$

where the coefficients r_0 and $\bar{r} = (r_y, r_x, r_z, r_w)^\intercal$ are:

$$
\begin{aligned}
r_0 &= -\theta\log\beta + (\theta-1)\left(\log k_1 + \left(k_1-1\right)A^\intercal\mu_X\right) + \mathcal{M}^\intercal\lambda - \frac{1}{2}\lambda^\intercal h\lambda, \\
\bar{r} &= (1-\theta)\left(k_1-1\right)A + \mathcal{K}^\intercal\lambda - \frac{1}{2}\lambda^\intercal H\lambda.
\end{aligned}
$$

Therefore, the risk-free rate is given by $r_t = r_0 + \bar{r}^\intercal X_t$ and the vector of risk prices is given by $\Omega(X_t) = (\Omega_y, \Omega_x, \Omega_z, \Omega_w)^\intercal = \Sigma(X_t)^\intercal\lambda$, where it turns out that $r_y = r_w = 0$ and $\Omega_y = \Omega_w = 0$.

Term Structures

Following Duffie et al. (2000), the risk-neutral dynamics of X_t are given by:

$$
\begin{aligned}
dX_t &= \left(\mathcal{M}^{\mathcal{Q}} + \mathcal{K}^{\mathcal{Q}}X_t\right)dt + \Sigma(X_t)d\mathcal{B}_t^{\mathcal{Q}}, \\
\mathcal{M}^{\mathcal{Q}} &= \mathcal{M} - h\lambda, \\
\mathcal{K}^{\mathcal{Q}} &= \mathcal{K} - H\lambda, \\
d\mathcal{B}_t^{\mathcal{Q}} &= d\mathcal{B}_t + \Sigma(X_t)^\intercal\lambda\, dt.
\end{aligned}
$$

[26]Note that the above system of equations could yields multiple solutions. Tauchen (2011) proposes to select the root which ensures the non-explosiveness of the system. Alternatively, one could select an economically reasonable solution.

Then, we can compute the discounted value of several payouts, such as $D_{t+\tau}, D^{\varphi}_{t+\tau}$ and the unitary payout of a risk-less bond:

$$\mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t}\exp(v^{\intercal}X_{t+\tau})\right] = \mathbb{E}^{\mathcal{Q}}\left[\exp(-\int_0^{\tau} r_{t+s}ds + v^{\intercal}X_{t+\tau})\right] = \exp(q_0(\tau) + q(\tau)^{\intercal}X_t),$$

where $v \in \{v_D, v_{\varphi}, (0,0,0,0)^{\intercal}\}$. The deterministic function $q_o(\tau)$ and $q(\tau) = (q_y(\tau), q_x(\tau), q_z(\tau), q_w(\tau))$ solve the following system of ODEs:

$$\begin{aligned} q_0'(\tau) &= -r_0 + \left(\mathcal{M}^{\mathcal{Q}}\right)^{\intercal}q(\tau) + \frac{1}{2}q(\tau)^{\intercal}h\,q(\tau), \\ q'(\tau) &= -\bar{r} + \left(\mathcal{K}^{\mathcal{Q}}\right)^{\intercal}q(\tau) + \frac{1}{2}q(\tau)^{\intercal}Hq(\tau), \end{aligned}$$

with initial conditions $q_0(0) = 0$ and $q(0) = v$.

Therefore, the risk-less bond price, the strip price of the aggregate payout and the strip price of the cross-sectional payout are given by

$$\begin{aligned} B_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^{\intercal}X_t), \quad \text{with} \quad v = (0,0,0,0)^{\intercal}, \\ P_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^{\intercal}X_t), \quad \text{with} \quad v = v_D, \\ P^{\varphi}_{t,\tau} &= \exp(q_0(\tau) + q(\tau)^{\intercal}X_t), \quad \text{with} \quad v = v_{\varphi}. \end{aligned}$$

Aggregate Equity

The price P_t of the stock paying out the aggregate payout D_t can be computed either as the time integral of the corresponding strip price over any maturity or via an exponential affine approximation. Such exponential affine approximation is given by

$$P_t = D_t \exp(A_0^m + (A^m)^{\intercal}X_t),$$

where the coefficients A_0^m, $A^m = (A_y^m, A_x^m, A_z^m, A_w^m)^{\intercal}$, and the endogenous constant k_1^m solve the following system:

$$\begin{aligned} 0 =& (\theta-1)(k_1-1)A + (k_1^m - 1)A^m + \mathcal{K}^{\intercal}\chi_m + (1/2)\chi_m^{\intercal}H\chi_m, \\ 0 =& \theta\log\beta - (\theta-1)(\log k_1 + (k_1-1)A^{\intercal}\mu_X) - (\log k_1^m + (k_1^m-1)(A^m)^{\intercal}\mu_X) + \mathcal{M}^{\intercal}\chi_m \\ &+ (1/2)\chi_m^{\intercal}h\chi_m, \\ 0 =& A_0^m + (A^m)^{\intercal}\mu_X - \log k_1^m + \log(1-k_1^m), \end{aligned}$$

where $\chi_m = v_D + k_1^m A^m - \lambda$. It turns out that $A_y^m = A_w^m = 0$. Therefore, the risk premium on aggregate equity is equal to $RP^m(x_t) = ((A^m + v_D)^{\intercal}\Sigma(X_t))\Omega(X_t)$, which is an affine function of x_t only.

Cross-Sectional Equity

The price of the stock paying out the stream D_t^{φ} can be computed either as the time integral of the corresponding strip price over any maturity or via an exponential affine approximation.

Such exponential affine approximation is given by

$$P_t^\varphi = D_t^\varphi \exp(A_0^\varphi + (A^\varphi)^\intercal X_t),$$

where the coefficients A_0^φ, $A^\varphi = (A_y^\varphi, A_x^\varphi, A_z^\varphi, A_w^\varphi)^\intercal$, and the endogenous constant k_1^φ solve the following system:

$$\begin{aligned}
0 =& (\theta - 1)(k_1 - 1)A + (k_1^\varphi - 1)A^\varphi + \mathcal{K}^\intercal \chi_\varphi + (1/2)\chi_\varphi^\intercal H \chi_\varphi, \\
0 =& \theta \log \beta - (\theta - 1)(\log k_1 + (k_1 - 1)A^\intercal \mu_X) - (\log k_1^\varphi + (k_1^\varphi - 1)(A^\varphi)^\intercal \mu_X) + \mathcal{M}^\intercal \chi_\varphi \\
& + (1/2)\chi_\varphi^\intercal h \chi_\varphi, \\
0 =& A_0^\varphi + (A^\varphi)^\intercal \mu_X - \log k_1^\varphi + \log(1 - k_1^\varphi),
\end{aligned}$$

where $\chi_\varphi = v_\varphi + k_1^\varphi A^\varphi - \lambda$. It turns out that $A_y^\varphi = A_w^\varphi = 0$. Therefore, the risk premium on the cross-sectional stock is equal to $RP^\varphi(x_t) = ((A^\varphi + v_\varphi)^\intercal \Sigma(X_t))\Omega(X_t)$, which is an affine function of x_t only.